I. Radiative and nonradiative transitions in semiconductors

(a) Radiative Transitions
(b) Traps
Non-radiative Transition

Luminescent Impurity Centers
Electron-hole Pairs
Radiative transitions in (a) direct and (b) indirect semiconductors. An additional wave vector, $k_i$ (shown by a dashed arrow), is required for a radiative transition in an indirect semiconductor. The additional momentum needed for such a transition has to come from lattice vibrations or from an impurity.

$$R_r = Bpn$$

$$\tau_r = \frac{1}{B_R N_A}$$
Nonradiative recombination processes, reduce the light emission efficiency. These nonradiative transitions can be characterized by a nonradiative lifetime of electron-hole pairs, $\tau_{nr}$.

The fraction of the electron-hole pairs that recombine emitting light is called the quantum efficiency (or radiative efficiency), $\eta_q$:

$$\eta_q = \frac{\tau_l}{\tau_r}$$

where

$$\tau_l = \frac{\tau_r \tau_{nr}}{\tau_r + \tau_{nr}}$$

is the overall lifetime.

The LED cutoff frequency:

$$f_T = \frac{1}{2 \pi \tau_l}$$
Lasers

\[ E_2 - E_1 = \hbar \omega \]

**Absorption**

\[ \frac{dN_2}{dt}_{spont} = -a_{12} N_2 \]

\( N_2 \) is the number of particles in state 2 and \( a_{12} \) is the spontaneous emission rate.

**Spontaneous Emission**

**Stimulated Emission**

\[ \frac{dN_2}{dt}_{induced} = -b_{12} f(\omega) N_2 \]

\[ \frac{dN_1}{dt}_{induced} = -b_{21} f(\omega) N_1 \]
The conditions for lasing

In the equilibrium:

\[
\left( \frac{dN_2}{dt} \right)_{\text{spont}} + \left( \frac{dN_2}{dt} \right)_{\text{induced}} = \left( \frac{dN_1}{dt} \right)_{\text{induced}}
\]

\[
\begin{align*}
\left( \frac{dN_2}{dt} \right)_{\text{induced}} &= -b_{12} f(\omega) N_2 \\
\left( \frac{dN_1}{dt} \right)_{\text{induced}} &= -b_{21} f(\omega) N_1
\end{align*}
\]

\[b_{12} = b_{21} \text{ (Einstein, 1917)}\]

In equilibrium, according to the Boltzmann statistics

\[
\frac{N_2}{N_1} = \exp\left(-\frac{\Delta E_{21}}{k_B T}\right) << 1
\]

Therefore, close to equilibrium

absorption rate > emission rate

If \(N_2 > N_1\),

emission rate > absorption rate
Consider a medium with two energy states, $E_2$ and $E_1$, with $N_2$ particles per unit volume in state 2 and $N_1$ particles per unit volume in state 1.

The intensity of the electromagnetic wave, $I_\omega$, propagating in this medium in direction $x$ is governed by the following differential equation:

$$\frac{dI_\omega}{dx} = \gamma I_\omega$$

$$I_\omega = I_\omega(x = 0)\exp(\gamma x)$$

$$\gamma = K(N_2 - N_1)$$

When $N_2 < N_1$, $\gamma < 0$ (absorption)

When $N_2 > N_1$, $\gamma > 0$ (gain)

The condition $N_2 > N_1$ is called *population inversion*.
In a simple two-level system it is difficult to achieve the population inversion

\[ \exp (\varepsilon / k_B T) \]

Equilibrium

Non-equilibrium

Pumping is used to achieve \( N_2 > N_1 \)

However, since \( b_{21} = b_{12} \), even at very high pumping

level \( N_2 \sim N_1 \)
Population inversion in semiconductors

The photon with frequency \( \nu \) (light wavelength \( \lambda = \frac{c}{\nu} \)) has an energy \( E_{ph} = h\nu \). When the semiconductor is shined with the photon flux with energy \( E_{ph} > E_g \) BOTH the absorption and the emission occur.
Conditions for population inversion in semiconductors

**The probability of photon ABSORPTION** \( (E_V \Rightarrow E_C) \) is proportional to

1) \( N_1 \) - the number of electrons with energy \( E_V \);
2) \( P_2 \) - the number of holes (empty positions) with energy \( E_C \);

**The probability of photon EMISSION** \( (E_C \Rightarrow E_V) \) is proportional to

1) \( N_2 \) - the number of electrons with energy \( E_C \);
2) \( P_1 \) - the number of holes with energy \( E_V \);
The number of electrons with the energy $E$ is proportional to

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

The number of holes with the energy $E$ is proportional to $f_p = 1 - f(E)$;
\[ N_1 \sim f_1 = f(E_v); \]
\[ N_2 \sim f_2 = f(E_c); \]
\[ P_1 \sim f_{p1} = 1-f_1; \]
\[ P_2 \sim f_{p2} = 1-f_2; \]

The ST. EMISSION INTENSITY is proportional to
\[ f_2 \times f_{p1} = f_2 \times (1-f_1); \]

The ABSORPTION INTENSITY is proportional to
\[ f_1 \times f_{p2} = f_1 \times (1-f_2); \]

The absorption coefficient
\[ \alpha = \text{ABSORPTION RATE} - \text{ST. EMISSION RATE} \]
\[ \alpha \sim f_1 \times (1-f_2) - f_2 \times (1-f_1) = (f_1 - f_2); \]

In equilibrium, \( \alpha > 0 \)
The condition for the light emission is \( \alpha < 0 \), or

\[
f_{p1} + f_2 = f_p(E_v) + f_n(E_c) > 1;
\]

\[
f_n = \frac{1}{1 + \exp \left( \frac{E - E_{Fn}}{k_B T_e} \right)}
\]

\[
f_p = \frac{1}{1 + \exp \left( \frac{E_{fp} - E}{k_B T_p} \right)}
\]

We are trying to achieve a stimulated emission of a photon with the energy \( E_g \).

For \( \alpha (E_g) < 0 \), \( E_{fn} - E_{fp} > E_g \)

*The Quasi-F. levels must be inside the “C” and the “V” bands*
Quantum well heterostructure laser

(a) Zero bias

(b) Forward bias
Quantum well heterostructure laser