If the sample parameters do not meet the Kroemer criteria, 

\[ n_0L > (n_0L)_1 = \frac{3\epsilon v_s}{q|\mu_d|} \]

the sample is stable, i.e. the high-field domains do not form. However, the differential mobility is still negative at high electric fields. In this case, 

1. What is the I-V characteristic of the sample? 
2. How does the negative mobility change the device impedance?
12 Gunn Effect Amplifiers

I. Stable Gunn Diodes: I-V Characteristics

Regular “ohmic” n-sample with n+ contacts.

An average field in the sample:

\[ F_0 = \frac{U}{L} \]

The current density in the sample is constant:

\[ j = qn(x)\mu E(x) = \text{const} \]

Therefore, when \( n(x) \) decreases, \( F(x) \) increases
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I. Stable Gunn Diodes: I-V Characteristics

Expected I-V curve: \( I = qS n_0 \nu(F) \)

If the electric field is uniform, \( F = U/L \) and

\[ I = qS n_0 \nu(U/L) \]
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I. Stable Gunn Diodes: I-V Characteristics

Neglecting the diffusion and correspondingly the boundary condition at the anode contact, the field distribution can be obtained from Poisson and current continuity equations:

1: \( \frac{U}{L} < F_p \)

2: \( \frac{U_1}{L} > F_p \)

3: \( \frac{U_2}{L} > F_p \)

\[ \frac{dF}{dx} = \frac{q(n_0 - n)}{\varepsilon} \]

\[ j = qn(x)\mu E(x) = \text{const} \]
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I. Stable Gunn Diodes: I-V Characteristics

If $U/L > F_p$, the field and electron concentration distributions near the cathode will be similar to the distributions discussed above until the point $X_p$ is reached where $F = F_p$. Beyond this point the increase in the electric field leads to the decrease of the electron velocity due to the negative differential mobility. As a result the electron concentration has to increase even more to ensure the constant current density along the sample. This increase results in a large field derivative $\frac{dF}{dx}$,

$$\frac{dF}{dx} = \frac{q(n_0 - n)}{\varepsilon}$$

The increase in the electric field leads to a bigger drop in the electron velocity and the field distribution becomes even steeper closer to the anode.

When $F > F_p$, a small increase in the current requires very large increase in the applied voltage.

There is no NDR region in the I-V curve.

1: $U/L < F_p$
2: $U_1/L > F_p$
3: $U_2/L > F_p$
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II. Stable Gunn Diodes: device impedance

As follows from the above discussion the dc differential resistance is positive at $U > U_p$ owing to the electron injection from the cathode. However, the electron distribution can follow the variation in the external voltage only at frequencies smaller than the transit frequency

$$f_T = v_s/L$$

At frequencies close to $f_T$ and higher the injection of electrons from the cathode will not be able to prevent the growth of a small fluctuation moving from the cathode to the anode and the device may still exhibit the negative differential resistance and can be used as an amplifier.
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II. Stable Gunn Diodes: device impedance

To find the device impedance we will solve two equations:

\[
\frac{\partial F}{\partial x} = \frac{q(n_0 - n)}{\varepsilon}
\]

\[
j = qnv(F) + qD(F) \frac{\partial n}{\partial x} + \varepsilon \frac{\partial F}{\partial t}
\]

One obvious solution to them is the “DC” one:

\[
F = \frac{U}{L}
\]

\[
n = n_0 = N_D - N_A
\]
II. Stable Gunn Diodes: device impedance

To find the device impedance we will seek the solution in the form of small sinusoidal signals:

\[ F = F_0 + F_1 e^{i(kx-\omega t)} \]
\[ n = n_0 + n_1 e^{i(kx-\omega t)} \]

\[ F_1 \ll F_0 \]
\[ n_1 \ll n_0 \]

Substituting \( F(x,t) \) and \( n(x,t) \) gives the following dispersion equation (i.e. the solution for non-zero \( n_1 \) and \( F_1 \))

\[ \frac{\partial F}{\partial x} = \frac{q(n_0 - n)}{\varepsilon} \]
\[ j = qn\nu(F) + qD(F) \frac{\partial n}{\partial x} + \varepsilon \frac{\partial F}{\partial t} \]

\[ \omega = -k\nu(F_0) - i \left[ \frac{1}{\tau_{md}(F_0)} + Dk^2 \right] \]

\[ \tau_{md} = \frac{\varepsilon}{qn_0 d\nu/dF} \]
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The wavelength (the characteristic size of the fluctuation) \( \lambda \) is related to the wave vector:

\[
k = \frac{2\pi}{\lambda}
\]

see:

\[
F = F_0 + F_1 e^{i(kx - \omega t)}
\]
\[
n = n_0 + n_1 e^{i(kx - \omega t)}
\]

\[
\omega = -kv(F_0) - i\left[\frac{1}{\tau_{md}(F_0)} + Dk^2\right]
\]

Frequency

\[
\text{Re}(\omega) = -kv(F_0)
\]

\[
\text{Im}(\omega) = -\frac{1}{\tau_{md}} - Dk^2
\]

Attenuation
12 Gunn Effect Amplifiers

\[ \text{Im}(\omega) = -\frac{1}{\tau_{md}} - Dk^2 \]

If \( \tau_{md} < 0 \) the fluctuation may grow

However, short wavelength fluctuation may decay due to diffusion

\[ 0 \leq k < \frac{1}{(D|\tau_{md}|)^{1/2}} \]

Neglecting diffusion, we can see how the fluctuation varies in space:

\[ \text{Re}(k) = -\frac{\omega}{v} \]
\[ \text{Im}(k) = -\frac{1}{\tau_{md}v} \]

The wave amplitude INCREASES towards the anode if \( \tau_{md} < 0 \)
Gunn-effect GaAs traveling wave amplifier
To fine the impedance let us again seek the small-signal solution neglecting diffusion:

\[-\varepsilon v \frac{\partial F}{\partial x} + qn_0 v + \varepsilon \frac{\partial F}{\partial t} = j\]

\[j = j_0 + j_1^0 e^{-i\omega t}\]

\[F = F_0 + F_1^0 (1 + a e^{ikx}) e^{-i\omega t}\]

At the cathode \((x=0)\), \(F(0) = F_0\)

Hence, \(a = -1\)

\[qn_0 v(F_0) = j_0\]

\[-ik = i \frac{\omega}{v(F_0)} - \frac{1}{\tau_{md} v(F_0)}\]

\[F_1^0 = \frac{j_1^0}{qn_0 \mu_d - i\varepsilon \omega}\]

It then follows from above
12 Gunn Effect Amplifiers

Small-signal impedance:

\[ Z(\omega) = \int_0^L F_1^0(1 - e^{ikx}) \, dx \]

Using

\[ F_1^0 = \frac{j_1^0}{qn_0\mu_d - i\varepsilon\omega} \]

we obtain:

\[ Z(\omega) = -\frac{L^2}{\varepsilon v S} \frac{e^{-\alpha} + \alpha - 1}{\alpha^2} \]

where

\[ \alpha = L\left(-\frac{1}{\tau_{mdv}} + i\frac{\omega}{v}\right) \]
Using

\[ Z(\omega) = -\frac{L^2}{\varepsilon v s} \frac{e^{-\alpha} + \alpha - 1}{\alpha^2} \]

\[ \alpha = L \left( -\frac{1}{\tau_{ma} v} + i \frac{\omega}{v} \right) \]

we can plot the diode impedance. However this expression does not account for (i) non-uniform field distribution and (ii) diffusion.

More accurate results
(dashed lines – neglecting the diffusion; solid lines – including the diffusion):
Is the sample stable?

\[ U_1^0 = Z(\omega)j_1^0 \]

If the diode is biased from the current source, then \( j_1 = 0 \).

\[ U_1 \neq 0 \text{ only if } Z(\omega) \rightarrow \infty \]

However, the impedance does not have poles \( (\alpha^2 \neq 0) \):

\[ Z(\omega) = -\frac{L^2}{\varepsilon \nu S} \frac{e^{-\alpha} + \alpha - 1}{\alpha^2} \]

where \[ \alpha = L \left( -\frac{1}{\tau_{md\nu}} + i\frac{\omega}{\nu} \right) \]

If the diode is biased from the voltage source, then \( U_1 = 0 \).

\[ j_1 \neq 0 \text{ only if } Z(\omega) = 0, \text{ or} \]

\[ e^{-\alpha} + \alpha - 1 = 0 \]
12 Gunn Effect Amplifiers

$$e^{-\alpha} + \alpha - 1 = 0$$

This equation can be solved numerically.

<table>
<thead>
<tr>
<th>m</th>
<th>$\alpha_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-2.09 \pm i.746$</td>
</tr>
<tr>
<td>2</td>
<td>$-2.69 \pm i.13.88$</td>
</tr>
<tr>
<td>3</td>
<td>$-3.03 \pm i.20.22$</td>
</tr>
<tr>
<td>4</td>
<td>$-3.22 \pm i.26.54$</td>
</tr>
<tr>
<td>5</td>
<td>$-3.50 \pm i.32.85$</td>
</tr>
</tbody>
</table>

When $\text{Re}(\alpha_1) > 2.09$ (see the table),
the diode becomes unstable when biased with the voltage source.

$$\alpha = L \left( -\frac{1}{\tau_{md} v} + i \frac{\omega}{v} \right)$$

$$\alpha_m = \frac{T_0}{\tau_{md}} - i2\pi f T_0$$

where $T_0 = L/v$

$$n_0 L < \frac{2.09 e v}{q |\mu_d|}$$

The Kroemer criterion

$$n_0 L \lesssim 6 \cdot 10^{11} \text{ cm}^{-2}$$
12 Gunn Effect Amplifiers

Traveling wave (active medium) propagation amplifier
12 Gunn Effect Amplifiers

Reflection type amplifier

\[ G = \left| \frac{Z(\omega) - Z_L(\omega)}{Z(\omega) + Z_L(\omega)} \right|^2 \]

If \( (Re(Z(\omega))) < 0 \)

the gain is bigger then unity.
12 Gunn Effect Amplifiers

Home work
(due Tuesday, March 2\textsuperscript{nd})

plot the impedance (real part and imaginary part) as a function of frequency using

\[ Z(\omega) = -\frac{L^2}{\varepsilon \nu S} \frac{e^{-\alpha} + \alpha - 1}{\alpha^2} \]

for the GaAs Gunn diode with \( L = 10 \ \mu m, S = 100x100 \ \mu m^2 \) and \( n_0 = 10^{14} \ \text{cm}^{-3} \)

The frequency range: 0…..40 GHz

Plot the Real and the Imaginary parts of the diode impedance using PARALLEL equivalent circuit (R connected in parallel with X).

Indicate the bias voltage needed to achieve the NDR regime.

Use \( \mu_d = -700 \ \text{cm}^2 /\text{V-s} \) and \( \nu_s \sim 10^7 \ \text{cm/s} \)