10 Universal HFET Model

Effect of Source and Drain Parasitic Resistances

The drain and source parasitic series resistances, $R_d$ and $R_s$, play an important role in limiting the device performance. These resistances can be accounted for by relating the intrinsic gate-source and drain-source voltages to their extrinsic (measured) counterparts, $V_{gs}$ and $V_{ds}$, by including the voltage drops across the series resistances:
\[ V_{GS} = V_{gs} - R_s I_d \]
\[ V_{DS} = V_{ds} - (R_s + R_d) I_d \]

\[ V_{xy} - \text{Intrinsic voltage} \]
\[ V_{xy} - \text{Measured voltage} \]

Combining this with “intrinsic” HFET model:

\[ I_d = \frac{W \mu n c_i}{L} \times \begin{cases} 
V_{GT} V_{DS} - V_{DS}^2 / 2, & \text{for } V_{DS} \leq V_{SAT} \\
V_L^2 \left[ \sqrt{1 + (V_{GT} / V_L)^2} - 1 \right], & \text{for } V_{DS} > V_{SAT} 
\end{cases} \]

\[ V_{SAT} = V_{GT} - V_L \left[ \sqrt{1 + (V_{GT} / V_L)^2} - 1 \right] \]
Results in the following I-V model:

\[ I_{sat} = \frac{\beta V_{gt}^2}{1 + \beta R_s V_{gt} + \sqrt{1 + 2\beta R_s V_{gt} + \left(\frac{V_{gt}}{V_L}\right)^2}} \]

\[ V_{sat} = V_{gt} + \left( R_d - \frac{1}{\beta V_L} \right) I_{sat} \]

where \( \beta = W \mu_n c_i / L \) is the transconductance parameter.
Effective 2D gas “thickness” and 2D electron gas sheet density

**MOSFET**

\[ q n_s = C_i V_{gt} \]

\[ C_i = \frac{\varepsilon_i \varepsilon_o}{d} \]

d is the dielectric (SiO₂) thickness

**HFET**

\[ q n_s = C_i V_{gt} \]

\[ C_i = \frac{\varepsilon_i \varepsilon_o}{d + \Delta d} \]

d is the barrier (AlGaAs) thickness,

\( \Delta d \) is the effective thickness of the 2D gas layer
Effect of large positive gate bias – electron spill-over in HFETs

\[ n_s' = n_s \left(1 + \left(n_s/n_{max}\right)^\gamma\right)^{-\frac{1}{\gamma}}; \]

\[ q n_s' = C_i V_{gt} \]

\( \gamma \approx 3 \) and \( n_{max} \approx 7 \times 10^{11} \text{ cm}^{-2} \)

for AlGaAs/GaAs HFETs

\[ I_{sat} = \frac{g_{chi}V_{gt}}{1 + g_{chi}R_s + \sqrt{1 + 2g_{chi}R_s + \left(V_{gt}/V_L\right)^2}}; \]

\[ g_{chi} = \frac{q n_s W \mu_n}{L} \]
HFET Capacitances

The equivalent circuit corresponding to the intrinsic “two-diode” model.

\[
C_{GS} = \left. \frac{\partial Q_G}{\partial V_{GS}} \right|_{V_{GD},V_{GB}}, \quad C_{GD} = \left. \frac{\partial Q_G}{\partial V_{GD}} \right|_{V_{GS},V_{GB}},
\]

The charge under the gate

\[
Q_G \approx qW \int_0^L n_s dx = Wc_i \int_0^L [V_{GT} - V(x)] dx
\]
Below the saturation this integration results in

\[ Q_G = \frac{W^2\mu_n c_i^2}{I_d} \int_0^{V_{DS}} (V_{GT} - V)^2 dV = \frac{2}{3} C_i \frac{(V_{GS} - V_T)^3 - (V_{GD} - V_T)^3}{(V_{GS} - V_T)^2 - (V_{GD} - V_T)^2} \]

Where \( C_i = WL c_i \), and

\[ I_d = \frac{W\mu_n c_i}{L} \times \left[ V_{GT} V_{DS} - V_{DS}^2 / 2 \right], \quad \text{for } V_{DS} \leq V_{SAT} \]
HFET Capacitances

To find the differential capacitances we need to take the derivatives

\[ C_{GS} = \frac{\partial Q_G}{\partial V_{GS}} \bigg|_{V_{GD}, V_{GB}}, \quad C_{GD} = \frac{\partial Q_G}{\partial V_{GD}} \bigg|_{V_{GS}, V_{GB}}, \]

\[ Q_G = \frac{W^2 \mu_n C_i^2}{I_d} \int_0^{V_{DS}} (V_{GT} - V)^2 dV = \frac{2}{3} C_i \frac{(V_{GS} - V_T)^3 - (V_{GD} - V_T)^3}{(V_{GS} - V_T)^2 - (V_{GD} - V_T)^2} \]

\[ C_{GS} = \frac{2}{3} C_i \left[ 1 - \left( \frac{V_{GT} - V_{DS}}{2V_{GT} - V_{DS}} \right)^2 \right], \quad C_{GD} = \frac{2}{3} C_i \left[ 1 - \left( \frac{V_{GT}}{2V_{GT} - V_{DS}} \right)^2 \right] \]
HFET Capacitances

The capacitances in saturation are found by replacing $V_{DS}$ by $V_{SAT}$

$$C_{GSs} = \frac{2}{3} C_i; \quad C_{GDs} = 0$$
**HFET Capacitances**

Effect of the electron spill-over on the channel capacitance

\[
C_{ch} = WLq \frac{dn_s}{dV_{GS}} \approx \frac{C_i}{\left[1 + \left(\frac{n_s}{n_{max}}\right)^\gamma\right]^{1+1/\gamma}}
\]
The parasitic capacitances (marked by an additional subscript “p”) are:

\[ C_{ds,p} = C_{sd,p} = \frac{\varepsilon_s + \varepsilon_0}{2} W \times \frac{K(\sqrt{1 - k_{ds}^2})}{K(k_{ds})} \]

where \( k_{ds} = \frac{L_{DS}}{L_{DS} + 2L_S} \)

The accurate expressions for \( C_{gs,p} \) and \( C_{gd,p} \) are complicated because \( L_G \neq L_D \);
\( C_{gs,p} \) is not that important. However, \( C_{gd,p} \) is.
The only practical way is to use the expression for \( C_{ds,p} \) with \( L_{DS} \) replaced by \( L_{GD} \) and fit the data
HFET Cut-off Frequencies

Equivalent circuit of the HFET including parasitic elements.

The analysis of this circuit results in the following current gain:

\[ h'_{21} = -\frac{g_m - j\omega(R_S C_{gg,t} g_d + R_S C_{gd,t} g_m + g_m)}{j\omega[C_{gg,t} + (R_S + R_D) C_{gd,t} g_d + (R_S + R_D) g_m]} \]
HFET Cut-off Frequency

\[
h'_{21} = -\frac{g_m - j\omega(R_S C_{gg,t} g_d + R_S C_{gd,t} g_m + g_m)}{j\omega[C_{gg,t} + (R_S + R_D) C_{gd,t} g_d + (R_S + R_D) C_{gd,t} g_m]}
\]

Setting the condition \( h'_{21} = 1 \), we find \( f_T \) as:

\[
\frac{1}{2\pi f_T} = \frac{C_{gg,t}}{g_m} + \frac{C_{gg,t}}{g_m} (R_S + R_D) g_d + (R_S + R_D) C_{gd,t}
\]

Note, \( C_{gg} \) is the total gate capacitance, \( C_{gg} = C_{gs} + C_{gd} \).

In the saturation, \( C_{gg} \approx C_{gs} \).
HFET Maximum oscillation frequency

$f_{\text{max}}$ is the frequency where the *unilateral power gain* becomes equal to unity. Finding $f_{\text{max}}$ requires a complete equivalent circuit of the HFET.

Two mathematically identical “y” circuits are widely used
For the HFETs,

\[
\begin{align*}
y_{11} + y_{12} &= j\omega C_{gs} - j\omega C_{gd} = j\omega C_{gs} \\
y_{12} &= j\omega C_{gd} \\
y_{22} + y_{12} &= g_d + j\omega C_{dd} - j\omega C_{gd} = g_d + j\omega C_{sd} \\
y_{21} - y_{12} &= g_m - j\omega C_{dg} + j\omega C_{gd}
\end{align*}
\]
**HFET Maximum oscillation frequency**

\[
U = \frac{\|(y_{21} - y_{12})/\Delta y]\|^2}{4[\text{Re}(R_S + R_G + y_{22}/\Delta y)\text{Re}(R_S + R_D + y_{11}/\Delta y) - \text{Re}(R_S - y_{12}/\Delta y)\text{Re}(R_S - y_{21}/\Delta y)]}
\]

Setting \( U = 1 \),

\[
f_{\text{max}} = f_T \sqrt{8\pi R_G C_{gd,t} \left[ 1 + \left( \frac{2\pi f_T}{C_{gd,t}} \right) \Psi \right]}
\]

Where

\[
\Psi = (R_D + R_S) \frac{C_{gg,t}^2 g_d^2}{g_m^2} + (R_D + R_S) \frac{C_{gg,t} C_{gd,t} g_d}{g_m} + \frac{C_{gg,t} g_d}{g_m^2}
\]

Simplified expression for the \( f_{\text{max}} \):

\[
f_{\text{max}} = \frac{f_T}{2} \sqrt{\frac{R_{ds}}{R_g}}.
\]