BJT $I-V$ output characteristics. Also shown is a load line for the load resistance $R_L = 12.5 \text{ k}\Omega$ and two operating points corresponding to the base currents of 1\muA and 5\muA, respectively.
The common-emitter output characteristics of the same transistor with a superimposed small signal variation of the base current

The amplitude of the base current variation is 1 µA so that the base current changes sinusoidally in time from 1 µA to 3 µA. A small variation of the base current leads to a relatively large variation of the collector current (from 100 µA to 300 µA with the operating point at 200 µA) corresponding to a current gain of 100. In other words, the transistor operates as an amplifier for an ac signal.
The $h$-parameters are most frequently used for transistor characterization [at least at relatively low frequencies (below 100 MHz or so)] and are specified by transistor manufacturers in their data sheets.

\[ v_1 = h_{11}i_1 + h_{12}v_2 \]

\[ i_2 = h_{21}i_1 + h_{22}v_2 \]
The small signal parameters may be determined from different short-circuit or open-circuit measurements at the input and output ports:

\[
\begin{align*}
  h_{11} &= \frac{v_1}{i_1} | v_2 = 0 \\
  h_{12} &= \frac{v_1}{v_2} | i_1 = 0 \\
  h_{21} &= \frac{i_2}{i_1} | v_2 = 0 \\
  h_{22} &= \frac{i_2}{v_2} | i_1 = 0
\end{align*}
\]
07 High Frequency Parameters of BJTs and HBTs

SMALL SIGNAL EQUIVALENT CIRCUITS

\[ h_{11} = \frac{v_1}{i_1} \mid v_2 = 0 \]

\[ h_{12} = \frac{v_1}{v_2} \mid i_1 = 0 \]

\[ h_{21} = \frac{i_2}{i_1} \mid v_2 = 0 \]

\[ h_{22} = \frac{i_2}{v_2} \mid i_1 = 0 \]

The parameter \( h_{11} \) (\( h_i \)) is called the short-circuit input impedance, \( h_{12} \) is called the open-circuit reverse voltage ratio (\( h_r \)), \( h_{21} \) is called the short-circuit forward current ratio (\( h_f \)), and \( h_{22} \) is called the open-circuit output admittance (\( h_o \)).

\[ h_{fe} \] is equal to \( \beta \) and \( h_{fb} \) is equal to \( -\alpha \). For the other \( h \)-parameters, such relationships are more complicated.
High Frequency Parameters of BJTs and HBTs

Physical EQUIVALENT CIRCUITS: π-circuit

(a) Full and (b) simplified low-frequency hybrid-π equivalent circuits.

The transconductance, \( g_m \), is related to the dynamic (differential) resistance, \( r_e \), of the forward-biased emitter-base junction in the following way:

\[
g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}} \quad V_{th} = kT/q
\]

The resistance \( r_{bb'} \) is the base spreading resistance. The resistance \( r_{b'c} \) and the capacitance \( C_{b'c} \) in the hybrid-π equivalent circuit represent the dynamic (differential) resistance and the capacitance of the reverse-biased collector-base junction.
07 High Frequency Parameters of BJTs and HBTs

Physical EQUIVALENT CIRCUITS

\[ i_c \approx g_m v_{b'e} \]

\[ v_{b'e} \approx i_b r_{b'e} \]

\[ r_{b'e} \approx \frac{i_c}{i_b} \frac{1}{g_m} \approx \frac{hfe}{g_m} = \frac{\beta}{g_m} \]
Another useful equivalent circuit is a **T-equivalent circuit**

The emitter capacitance, $C_e$, is approximately equal to the sum of the diffusion capacitance of the emitter-base junction, $C_{ediff}$, and the depletion capacitance, $C_{ed}$. The resistance $r_c$ in the T-equivalent circuit describes the Early effect.
07 High Frequency Parameters of BJTs and HBTs

The relations between \( h \)-parameters and parameters of hybrid \( \pi \)-equivalent circuit.

<table>
<thead>
<tr>
<th>Definition of ( h )-parameters</th>
<th>( v_1 = h_{11}i_1 + h_{12}v_2 )  ( i_2 = h_{21}i_1 + h_{22}v_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic resistance of the forward-biased emitter-base junction</td>
<td>( r_e = \frac{\partial V_{b'e}}{\partial I_e} \approx \frac{V_{th}}{I_e} )</td>
</tr>
<tr>
<td>Transconductance</td>
<td>( g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}} )</td>
</tr>
<tr>
<td>Resistance in the hybrid-( \pi ) equivalent circuit</td>
<td>( r_{b'e} \approx \frac{i_c}{i_b} \frac{1}{g_m} \approx \frac{h_{fe}}{g_m} = \frac{\beta}{g_m} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( h )-parameter</th>
<th>Relation to parameters of hybrid ( \pi )-equivalent circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{oe} )</td>
<td>( 1/(r_{b'e} + r_{b'e}) + 1/r_{ce} + g_m r_{b'e} / (r_{b'e} + r_{b'e}) )</td>
</tr>
<tr>
<td>( h_{ie} )</td>
<td>( r_{bb'} + r_{b'e} r_{b'c} / (r_{b'e} + r_{b'c}) )</td>
</tr>
<tr>
<td>( h_{fe} )</td>
<td>( g_m r_{b'e} r_{b'c} / (r_{b'c} + r_{b'e}) )</td>
</tr>
</tbody>
</table>
Small-signal operation of BJT

In a typical circuit, the transistor operating point is chosen using **biasing resistors** (resistors $R_{B1}$ and $R_{B2}$). The base dc voltage (with respect to the ground) is equal to $V_{cc}R_{B1}/(R_{B1} + R_{B2})$ if the common-emitter gain, $\beta$, is large. The operating point also depends on the **emitter series resistance**, $R_E$. The purpose of this resistance is to make the operating point more stable with respect to temperature variations.

Coupling capacitances ($C_1$ and $C_2$) are used to connect the transistor stage to the rest of the circuit and to isolate the dc bias and ac signal. A bypass capacitor, $C_E$, shunts the resistance, $R_E$, for the ac signal.
Small-signal operation of BJT

The limitations of BJT operating frequencies imposed by time delays related to $RC$ time constants of a BJT equivalent circuit.

Using the T-equivalent circuit for the common-base configuration:

\[
i_e = v_{b'e \left(1 + j\omega C_e r_e \right)}/r_e = i_{eo \left(1 + j\omega C_e r_e \right)}
\]

where $i_{eo}$ is the ac current through resistance $r_e$

Hence, the common-base current gain $\alpha_\omega$ at the frequency $\omega$ is given by

\[
\alpha_\omega = i_c / i_e = i_c \left[i_{eo \left(1 + j\omega C_e r_e \right)}\right] = \alpha / \left(1 + j\omega C_e r_e \right)
\]

where $\alpha$ is the common-base current gain at $\omega = 0$

\[
\alpha_\omega = \alpha / \left(1 + j\omega / \omega_\alpha \right)
\]

**alpha cutoff frequency:**

\[
\omega_\alpha = 2\pi f_\alpha = 1/(C_e r_e)
\]
07 High Frequency Parameters of BJTs and HBTs

Small-signal operation of BJT

The common-emitter configuration

\[ i_b = v_{b'e} \left[ g_{b'e} + j\omega(C_{b'e} + C_{b'c}) \right] \]

where \( g_{b'e} = \frac{1}{r_{b'e}} = \frac{h_f}{r_e} \)

As shown before,

\[ r_{b'e} \approx \frac{i_c}{i_b} \approx \frac{h_f}{g_m} = \frac{\beta}{g_m} \]

Since \( i_c = g_m v_{b'e} \),

\[ \beta_\omega = \frac{i_c}{i_b} = \frac{g_m}{g_{b'e} + j\omega(C_{b'e} + C_{b'c})} = \frac{\beta}{\left(1 + j\omega/\omega_\beta\right)} \]

The beta cutoff frequency: \( \omega_\beta = 2\pi f_\beta = g_{b'e} / (C_{b'e} + C_{b'c}) \)
07 High Frequency Parameters of BJTs and HBTs

Comparison of \( f_\alpha \) and \( f_\beta \)

\[
\omega_\beta = 2\pi f_\beta = \frac{g_{b'e}}{C_{b'e} + C_{b'c}}
\]

\[
f_\beta = \frac{g_m}{2\pi h_{fe}(C_{b'e} + C_{b'c})}
\]

Since, in most cases, \( C_{b'e} \gg C_{b'c} \),

\[
f_\beta \approx \frac{f_\alpha}{h_{fe}}
\]
07 High Frequency Parameters of BJTs and HBTs

The **cutoff frequency**, \( f_T \), is defined as the frequency at which the magnitude of the short-circuit common-emitter current gain equals unity, that is, \( |\beta_\omega| = 1 \).

\[
f_T = f_\beta \sqrt{\beta^2 - 1} \approx f_\beta h_{fe} \approx \frac{g_m}{2\pi(C_{b'e} + C_{b'c})}
\]

where \( \beta = h_{fe} \) is the low-frequency, short-circuit, common-emitter current gain. Since usually \( C_{b'e} \gg C_{b'c} \),

\[
f_T \approx f_\alpha.
\]

At frequencies much larger than \( f_\beta \) but much smaller than \( f_T \),

\[
\beta_\omega \approx \frac{\beta_\omega}{(j\omega)} \quad \text{Since} \quad \beta_\omega = \frac{\beta}{1 + j\omega / \omega_\beta}
\]

\[
f_T \approx f_\beta h_{fe} \approx f |\beta_\omega|
\]

This equation can be used for deducing \( f_T \) from the measured values of \( \beta_\omega \) at high frequencies.
07 High Frequency Parameters of BJTs and HBTs

For a more accurate calculation of $f_T$ we have to account for additional time delays that are not represented by in the simple equivalent circuit.

One such delay is associated with the collector depletion layer transit time

$$\tau_{cT} \approx \frac{x_{dcb}}{v_{sn}}$$

where $x_{dcb}$ is the width of the collector-base depletion region and $v_{sn}$ is the electron saturation velocity (for $n$-$p$-$n$ transistors).

Another important delay is associated with the collector charging time, $\tau_c$, related to the collector series resistance, $r_{cs}$,

$$\tau_c = r_{cs} C_{b'c}$$

Finally, a parasitic capacitance, $C_p$, should be added to the collector capacitance, $C_{b'c}$.

$$f_T \approx \frac{1}{2\pi \tau_{\text{eff}}} \quad \tau_{\text{eff}} = \tau_e + \tau_c + \tau_{cT}$$

$$\tau_e = \frac{C_{b'e} + C_{b'c} + C_p}{g_m} \approx \frac{(C_{b'e} + C_{b'c} + C_p) V_{th}}{I_c}$$
07 High Frequency Parameters of BJTs and HBTs

\[ f_T \approx \frac{1}{2\pi \tau_{\text{eff}}} \]
\[ \tau_{\text{eff}} = \tau_e + \tau_c + \tau_{cT} \]

\[ \tau_e = \frac{C_{b'e} + C_{b'c} + C_p}{g_m} \approx \frac{(C_{b'e} + C_{b'c} + C_p) V_{th}}{I_c} \]

Where is the base transit time, \( W^2/(2D_n) \)?

In most transistors, \( \tau e \) is an important or even the dominant contribution to the total delay time.

This time can be reduced by increasing the collector current.
07 High Frequency Parameters of BJTs and HBTs

Maximum oscillation frequency

The frequency at which the power gain of the transistor is equal to unity under optimum matching conditions for the input and output impedances is called the **maximum oscillation frequency**, $f_{\text{max}}$.

![Circuit Diagram]

Using a simplified $\pi$-equivalent circuit where we neglect $r_{b'c}$ and $r_{ce}$,

$$f_{\text{max}} = \sqrt{\frac{f_T}{8\pi r_{bb'} C_{b'c}}}$$

This result points to the very important role played by the base spreading resistance.
07 High Frequency Parameters of BJTs and HBTs

The summary of important equations related to BJT frequency response

<table>
<thead>
<tr>
<th>Common-base current gain</th>
<th>$\alpha_\omega = \alpha/(1 + j \omega/\omega_\alpha)$ where $\omega_\alpha = 2\pi f_\alpha = 1/(C_e r_e)$</th>
</tr>
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<tr>
<td>Common-emitter current gain</td>
<td>$\beta_\omega = \beta/(1 + j \omega/\omega_\beta)$ where $\omega_\beta = 2\pi f_\beta = g_{be}/(C_{be} + C_{bc})$</td>
</tr>
<tr>
<td>Cutoff frequency</td>
<td>Crude estimate: $f_T \approx \frac{g_m}{2\pi(C_e + C_{b'c})} \approx \frac{g_m}{2\pi C_e}$</td>
</tr>
<tr>
<td></td>
<td>More accurate equation: $f_T \approx \frac{1}{2\pi\tau_{eff}}$ where $\tau_{eff} = \tau_e + \tau_c + \tau_{cT}$,</td>
</tr>
<tr>
<td></td>
<td>$\tau_e = (C_e + C_{b'c} + C_p) g_m$, $\tau_{cT} \approx x_{dc} v_{sn}$, $\tau_c = r_{cs} C_{b'c}$,</td>
</tr>
<tr>
<td>Maximum oscillation frequency, $f_{max}$</td>
<td>$f_{max} = \frac{f_T}{\sqrt{8\pi r_{bb'} C_{b'c}}}$</td>
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