Real p-n junction diode I-V characteristics

Consider a diode rectifier rated for 1000 V reverse bias.

**diode base:** n-layer, thick and low-doped to absorb high voltage
Design the required thickness of the diode base (n-layer)

Plot the dependence $F_m (N_D)$ at $V_r = -1000$ V

\[ F_m = \sqrt{\frac{2qN^* (V_{bi} - V_r)}{\varepsilon \varepsilon_0}} \]
Design the required thickness of the diode base (n-layer)

Plot the dependence $F_m (N_D)$ at $V_r = -1000$ V

The doping in the n-type material must not exceed $\sim 5\times 10^{14}$ cm$^{-3}$
Find the depletion region width at $N_D = 5 \times 10^{14} \text{ cm}^{-3}$ and $V_r = -1000 \text{ V}$

The thickness of n-type material must not be less than $\sim 50 \mu\text{m}$
Now consider the diode at zero or positive bias:
The depletion region width $W(V=0) \approx 0$.
Therefore, the equivalent circuit of the diode at $V \geq 0$ is:

If the p-layer is highly doped, it can be made very thin $\Rightarrow R_{sp} \approx 0$
Let us find the resistance $R_{sn}$
Assume the device area is 1mm x 1 mm (power device)

$n$-type floating metal contact

$R_{sp}$

$n$-type floating metal contact

$d_n = 50 \mu m$

$N_D = 5 \times 10^{14} \text{ cm}^{-3}$

$\mu_n = 1000 \text{ cm}^2/(\text{V*s})$

$d_n = 50 \mu m$

$A = 1 \text{ mm}^2$

$\sigma = q n \mu_n = 0.08 \text{ S/cm}$

$R_{sn} = (1/\sigma)d_n/A = 6.25 \text{ Ohm}$
Diode I-V characteristics

Ideal diode

“Real” diode with 50 um base

This line does NOT represent real pn-diode I-V because it does not account for carrier injection at forward bias
p-n junction under forward bias

$V > 0$: the bias is called POSITIVE or Forward, if the polarity is opposite to the built-in barrier.

Typical notation: $V_F$. Forward bias REDUCES the potential barrier

$V < 0$: the bias is called NEGATIVE or Reverse if the polarity is the same as the built-in barrier.

Typical notation: $V_R$. Reverse bias INCREASES the potential barrier
$V = 0; \text{ The electron concentration on the p-side: } n_p = n_n e^{-\frac{qV_{bi}}{kT}}$

Also, $n_p = n_{p0}$ (equilibrium electron concentration on the p-side) because the junction is in equilibrium

$V > 0; \text{ New barrier height } = qV_{bi} - qV.$

$$n_p = n_n e^{-\frac{qV_{bi} - qV}{kT}} = n_{p0} e^{\frac{qV}{kT}}$$

The concentration of injected electrons exponentially increases with the forward voltage.
Hole injection

In the equilibrium

$$\frac{p_n}{p_{p0}} = e^{-qV_{bi}/kT} \quad (1)$$

Under *forward bias* $V_F$, the barrier decreases by $qV_F$:

$$\frac{p_n}{p_{p0}} = e^{-q(V_{bi} - V)/kT} \quad (2)$$

Dividing (2) by (1):

$$\frac{p_n}{p_{n0}} = e^{qV/kT} \quad (3) \quad p_n = p_{n0} e^{qV/kT}$$

The concentration of injected holes exponentially increases with the forward voltage.
Summary of injected electron and hole concentrations

Injected electron and hole concentrations:

\[ n_p = n_{p0} \times e^{qV/kT} \]
\[ p_n = p_{n0} \times e^{qV/kT} \]

The change in the electron concentration in the p-region:

\[ \Delta n_p = n_p - n_{p0} = n_{p0} \left( \frac{e^{qV/kT}}{e^{qV/kT}} - 1 \right) \]

The change in the hole concentration in the n-region:

\[ \Delta p_n = p_n - p_{n0} = p_{n0} \left( \frac{e^{qV/kT}}{e^{qV/kT}} - 1 \right) \]
Carrier injection and forward current mechanism in p-n junction

- Forward bias lowers the potential barrier between n- and p-sides.
- Electrons and holes diffuse through the lowered barrier into p- and n-sides.
- Significant amount of extra electrons and holes gets injected into p- and n-sides correspondingly.
- Injected electrons recombine with holes in p-type material; injected holes recombine with electrons in n-type material.
- To compensate the loss of carriers due to recombination, additional electrons and holes are supplied by the voltage source through the contacts – *the forward current flows through the p-n junction.*
Note that the carrier injection takes place OUTSIDE the space-charge region of the p-n junction.

We assume that the space charge region is thin so that hot carriers quickly drift through the depletion region due to high initial velocity.

Outside the depletion region the field is low. Therefore the electron and hole transport can only be due to diffusion.
The injected carriers **diffuse** into n- and p-portions of the junctions and RECOMBINE along with DIFFUSION;
**Injected carrier spatial distribution (3)**

The steady-state distribution of the injected holes can be obtained from the following balance equation:

\[ \frac{\partial p}{\partial t} = \{\text{Income}\} - \{\text{Outcome}\} = 0, \text{ or} \]

\[ - \frac{p_n - p_{n0}}{\tau_p} + D_p \frac{\partial^2 p_n}{\partial x^2} = 0 \]

We define \( \delta p_n = p_n - p_{n0} \) which is the **excessive** hole concentration.

The solution is:

\[ \delta p_n (x) = \delta p_n (0) e^{-x/L_p} \]

where

\[ L_p = \sqrt{\tau_p D_p} \]

is the diffusion length of the holes.

Similarly, for injected electrons:

\[ \delta n_p (x) = \delta n_p (0) e^{-|x|/L_n} \]

where \(|x|\) is the absolute distance

\[ L_n = \sqrt{\tau_n D_n} \]

is the diffusion length of the electrons.
This solution describes non-uniform spatial distribution of the injected carriers:

\[ \delta p_n(x) = \delta p_n(0) e^{-x/L_p} \]

The solution applies to the regions that begin at the space-charge region edges.

Thus, \( p_n(0) \) is measured at \( x = x_n \)

Typically, for Si: \( L_n \approx 500 \, \mu m; \ L_p = 200 \, \mu m \)
**Injected carrier voltage and distance dependences**

As a function of distance from the depletion region edges, the injected carrier concentration can be found as

\[
p_n(x) = p_{n0} + p_{n0} \left( e^{qV / kT} - 1 \right) \times e^{-(x-x_n)/L_p}
\]

\[
n_p(x) = n_{p0} + n_{p0} \left( e^{qV / kT} - 1 \right) \times e^{-(|x|-x_p)/L_n}
\]

Injected (excessive) electron and hole concentrations:

\[
\Delta p_n(x) = p_{n0} \left( e^{qV / kT} - 1 \right) \cdot e^{-(x-x_n)/L_p}
\]

\[
\Delta n_p(x) = n_{p0} \times \left( e^{qV / kT} - 1 \right) \times e^{-(|x|-x_p)/L_n}
\]

\((x-x_n)\) and \((|x|-x_p)\) are the absolute distances counted from the edges of the space charge regions on the n- and p- sides correspondingly.
Forward current of the p-n junction

Injected (excessive) electron and hole concentrations in the p-n junction:

$$\Delta p_n(x) = p_n0(e^{qV/kT} - 1) \cdot e^{-(x-x_n)/L_p}$$

$$\Delta n_p(x) = n_p0 \times (e^{qV/kT} - 1) \times e^{-(|x|-x_p)/L_n}$$

Total excessive injected charges (A=junction area):

$$\Delta Q_n = q \times A \times \int_0^\infty \Delta n_p(x) \, dx = q \times A \times n_p0 \times \left(e^{qV/kT} - 1\right) \times L_n$$

$$\Delta Q_p = q \times A \times \int_0^\infty \Delta p_n(x) \, dx = q \times A \times p_n0 \times \left(e^{qV/kT} - 1\right) \times L_p$$

The charge $\Delta Q_n$ recombines at the rate of $\Delta Q_n/\tau_n$

The charge $\Delta Q_p$ recombines at the rate of $\Delta Q_p/\tau_p$

The current supplied from the contacts to compensate the recombination:

$$I = \Delta Q_n/\tau_n + \Delta Q_p/\tau_p$$
Forward current of the p-n junction (2)

\[ \Delta Q_n = q \times A \times n_{p0} \times \left( e^{qV/kT} - 1 \right) \times L_n \]
\[ \Delta Q_p = q \times A \times p_{n0} \times \left( e^{qV/kT} - 1 \right) \times L_p \]

The compensating current:

\[ I = \Delta Q_n / \tau_n + \Delta Q_p / \tau_p \]

\[ I = q \times A \times n_{p0} \times \left( e^{qV/kT} - 1 \right) \times L_n / \tau_n + q \times A \times p_{n0} \times \left( e^{qV/kT} - 1 \right) \times L_p / \tau_p \]

Electron current

Hole current

\[ I = q \times A \times \left( e^{qV/kT} - 1 \right) \times \left( n_{p0} \times \frac{L_n}{\tau_n} + p_{n0} \times \frac{L_p}{\tau_p} \right) \]

\[ L_{n,p} = \sqrt{D_{n,p} \tau_{n,p}} \quad \Rightarrow \quad \tau_{n,p} = L_{n,p}^2 / D_{n,p} \]

\[ I = q \times A \times \left( e^{qV/kT} - 1 \right) \times \left( n_{p0} \times \frac{D_n}{L_n} + p_{n0} \times \frac{D_p}{L_p} \right) \]
Total (e- and h-) p-n junction forward current

\[ I = q \times A \times \left( e^{qV/kT} - 1 \right) \times \left( n_{p0} \frac{D_n}{L_n} + p_{n0} \frac{D_p}{L_p} \right) \]

This current can be re-written as:

\[ I = I_S \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] \quad \text{where} \quad I_S = q A \left( \frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n} \right) \]
P-n junction current – voltage characteristic (I-V)

\[ I = I_S \left[ \exp\left( \frac{qV}{kT} \right) - 1 \right] \]

Forward current

Forward current

\[ I = I_{\text{gen.}}(e^{qV/kT} - 1) \]
Reverse current of the p-n junction

is given by the same expression as for the forward current

\[ I = I_S \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] \]

note that

\[ \exp \left( \frac{qV}{kT} \right) \rightarrow 0 \quad \text{when } V < 0 \text{ and } |V| \gg kT/q \approx 0.026 \text{ V} \]

Under typical reverse bias, \(|V_R| >> kT/q|:

\[ I \approx -I_S \]

\[ I_S = qA \left( \frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n} \right) \]
Reverse current of the p-n junction

\[ I = I_S \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] \quad \Rightarrow \quad I \approx -I_S \quad \text{at negative } V \]
The above I-Vs did NOT account for the injected carrier profile. Injected holes significantly increase the electron-hole pair concentration in the base:

Parameters used: $V = 0.7 \text{ V}$; $L_p = 200 \mu\text{m}$
Diode I-V characteristics

- Ideal diode
- Real diode w 50 um base (including injected carriers)
- Real diode w 50 um base (ignoring injected carriers)