p- n diode rectifiers
p-n diode rectifiers

\[
V = \exp\left(-\frac{kT}{qV_I} I_S\right)
\]

Time
Current

\(I_r = 0\) in an ideal rectifier.
\(V_F = 0\) in an ideal rectifier.

\(V_D = V_{Source}\)
Lecture objective: p-n diode at reverse bias

The diode $i$-$v$ relationship with some scales expanded and others compressed in order to reveal details.
Simulated Electron - Hole profile in Si p-n junction

Electron-hole concentration in a Si p-n junction.

Acceptor density $N_a = 5 \times 10^{15}$ cm$^{-3}$
Donor density $N_d = 1 \times 10^{15}$ cm$^{-3}$.

$T=300$K.

Dashed line show the boundaries of the depletion region
Charge distribution in the depletion region.

Total charge density in the semiconductor: \( \rho = q\times(N_d + p - N_a - n); \)

- \( p_n \approx 0; \)
- \( n_p \approx 0; \)
- \( \rho \approx -q\times N_A; \)
- \( n_n \approx 0; \)
- \( p_n \approx 0; \)
- \( \rho \approx +q\times N_D; \)
Mobile Charge density distribution in the p-n junction

Charge density (C/cm$^3$)

Depletion region

Free electrons

Holes

p-type

n-type

$p_n \approx 0$;
$n_p \approx 0$;

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Charge distribution in the depletion region.

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\[ p_n \approx 0; \]
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\[ \rho \approx 0; \]
\[ n_n \approx 0; \]
\[ p_n \approx 0; \]
\[ \rho \approx +q N_D; \]
Electric field profile in the p-n junction

In the following material, the electric field is denoted as “F”
(to distinguish from “E” = electron/hole energy).

The electric field distribution is defined by Poisson’s equation:

\[ \frac{dF}{dx} = \frac{\rho}{\varepsilon \varepsilon_0} = \frac{\rho}{\varepsilon_s}; \]

where \( \varepsilon_s = \varepsilon \varepsilon_0 \), \( \rho \) is the charge density

\( \varepsilon_0 \) – the dielectric permittivity of vacuum;
\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \times 10^{-14} \text{ F/cm} \)
\( \varepsilon \) – relative dielectric permittivity of the material
In Si, \( \varepsilon \approx 11.7 \)
Charge, Field, Potential Profiles in the p-n junction

\[ \frac{dF}{dx} = \frac{\rho}{\varepsilon_s}; \]
Charge, Field, Potential Profiles in the p-n junction

\[ \frac{dF}{dx} = \frac{\rho}{\varepsilon_s}; \]

\[ \begin{cases} \frac{-qN_a}{\varepsilon_s}, & \text{for } -x_p < x < 0 \\ \frac{qN_d}{\varepsilon_s}, & \text{for } 0 < x < x_n \end{cases} \]
Charge, Field,
Potential Profiles in the
p-n junction

\[
\frac{dF}{dx} = \frac{\rho}{\varepsilon_s};
\]

\[
dF \begin{cases} 
-\frac{qN_a}{\varepsilon_s}, & \text{for } -x_p < x < 0 \\
\frac{qN_d}{\varepsilon_s}, & \text{for } 0 < x < x_n
\end{cases}
\]

\[
F = \begin{cases} 
-F_m \left(1 + \frac{x}{x_p}\right), & \text{for } -x_p < x < 0 \\
-F_m \left(1 - \frac{x}{x_n}\right), & \text{for } 0 < x < x_n
\end{cases}
\]
Maximum Field in the p-n junction

From \( \frac{dF}{dx} = \frac{\rho}{\varepsilon_s} \):

On the p-side of the junction,

\[ F_m = \frac{qN_A}{\varepsilon\varepsilon_0} x_p \]

On the n-side of the junction,

\[ F_m = \frac{qN_D}{\varepsilon\varepsilon_0} x_n \]

At the junction interface: \( F_m (\text{p-side}) = F_m (\text{n-side}) \)

\[ F_m = \frac{qN_D}{\varepsilon\varepsilon_0} x_n = \frac{qN_A}{\varepsilon\varepsilon_0} x_p \]

\[ \frac{x_p}{x_n} = \frac{N_D}{N_A} \]
Voltage drop across the p-n junction

\[ F_m = \frac{qN_D}{\varepsilon \varepsilon_0} x_n = \frac{qN_A}{\varepsilon \varepsilon_0} x_p \]

The voltage drop across the p-n junction, \( V_{p-n} = -\int F(x) \, dx \)

\[ = -\{ \text{AREA UNDER the F(x) curve} \} \]

In equilibrium:
\[ V_{bi} = \frac{1}{2} F_m W \]

At reverse bias:
\[ V_{bi} - V_r = \frac{1}{2} F_m W \]
**Depletion region width in the p-n junction**

The total width of the depletion (space-charge) region.

From \( x_p = x_n \left( \frac{N_D}{N_A} \right) \)

\[
W = x_n + x_p = x_n \left( 1 + \frac{N_D}{N_A} \right);
\]

The voltage (the area of the triangle): \( V_{bi} - V_r = F_m \times W/2; \)

\[
F_m = \frac{qN_D}{\varepsilon \varepsilon_0} x_n = \frac{qN_A}{\varepsilon \varepsilon_0} x_p
\]

\[
V_{bi} - V_r = \left( \frac{qN_D}{\varepsilon \varepsilon_0} x_n \right) W/2 = \frac{qN_D}{2\varepsilon \varepsilon_0} W^2 \frac{1}{1 + N_D / N_A} = \frac{q}{2\varepsilon \varepsilon_0} W^2 \frac{N_D N_A}{N_A + N_D}
\]
Depletion region of the p-n junction summary

Depletion region (a.k.a space-charge region) width

\[
V_{bi} - V_r = \frac{q}{2\varepsilon\varepsilon_0} W^2 N'; \quad \left( \text{where } N' = \frac{N_D N_A}{N_A + N_D} \right)
\]

Depletion region width sharing between n- and p-sides:

\[
W = x_n + x_p
\]

\[
x_p = x_n \left( \frac{N_D}{N_A} \right)
\]

For strongly asymmetrical p-n junction:

\[
N_A \gg N_D, \quad V_{bi} - V_r = \frac{1}{2} \frac{q}{\varepsilon \varepsilon_0} N_D W^2 \quad \Rightarrow W \approx x_n
\]

\[
N_D \gg N_A, \quad V_{bi} - V_r = \frac{1}{2} \frac{q}{\varepsilon \varepsilon_0} N_A W^2 \quad \Rightarrow W \approx x_p
\]

The space-charge region is extended mainly to the low-doped side of the p-n junction.
Depletion region width as a function of voltage

From

\[ V_{bi} - V_r = \frac{q}{2\varepsilon\varepsilon_0} W^2 N' \]

The depletion region width \( W \):

\[ W = \sqrt{\frac{2\varepsilon\varepsilon_0 (V_{bi} - V_r)}{qN'}} \]

where

\[ N' = \frac{N_D N_A}{N_A + N_D} \]

For \( N_D \gg N_A \), \( N' = N_A \)

For \( N_A \gg N_D \), \( N' = N_D \)
Peak electric field in the p-n junction summary

Maximum electric field

\[ V_{bi} - V_r = \frac{1}{2} F_m W \]

\[ F_m = \sqrt{\frac{2q N'(V_{bi} - V_r)}{\varepsilon \varepsilon_0}} \]

\[ V_{bi} - V_r = \frac{\varepsilon \varepsilon_0 F_m^2}{2q N'} \]

\[ V_{max} = \frac{\varepsilon \varepsilon_0 F_{BD}^2}{2q N'} \]
The diode $i-v$ relationship with some scales expanded and others compressed in order to reveal details.
Mechanisms for breakdown
Two quantum processes give rise to breakdown:
(1) Impact ionization plus avalanche multiplication.
(2) Quantum tunneling (Zener effect).
Relative importance depends on doping level in the p- and n- regions:

![Graph showing avalanche and Zener effects](image.png)
Avalanche Breakdown mechanism

• Electric field accelerates electrons (and holes);
• The kinetic energy of electron in the electric field increases;
• Electron with high enough excessive energy can ionize the atom at collision.
• As a result, collision produces an electron-hole pair.
Avalanche Breakdown mechanism

Minimum additional electron energy required for ionization can be estimated using:

\[ \frac{m v^2}{2} \approx E_G \]

Additional electron energy comes from the electric field:

\[ \frac{m v^2}{2} \approx q F L_{FP} \]

where F is the electric field and \( L_{FP} \) is the “mean free path” – the average distance that the electron passes between collisions.

From this, the ionization condition: \( q F_{BD} L_{FP} \approx E_G \)

\( F_{BD} \) is the critical or “breakdown” field.

The ionization only occurs if electric field exceeds the critical value
In reverse-biased p-n junction, the electric field is concentrated within a narrow depletion region. The maximum electric field occurs exactly at the p-n interface. As the field $F_m$ exceeds the critical (breakdown) value:
Avalanche multiplication during the breakdown

Ionization coefficient $\alpha_e \approx \alpha_h$

Ionization coefficient $\alpha_e >> \alpha_h$
Multiplication and Ionization Coefficients

Consider uniform electric field in the avalanche region
(for simplicity)
The avalanche region width is $W$
Assume $\alpha_e = \alpha_h \approx \text{const (x)}$;

**Multiplication factor:**

\[
M = \frac{\text{Number of } e-h \text{ pairs at the output}}{\text{Number of } e-h \text{ pairs at the input}}
\]

\[
M = 1 + \alpha_e W + (\alpha_e W)^2 + (\alpha_e W)^3 + \cdots (\alpha_e W)^n + \cdots \\
= \frac{1}{1 - \alpha_e W} \quad \text{(for } \alpha_e W < 1) 
\]

The condition for avalanching: $\alpha_e W = 1$ (every carrier has one ionizing collision)
Avalanche breakdown interpretation using band diagrams

(1) Electrons from the p-side and holes from the n-side get accelerated in the depletion regions by strong electric field
Avalanche breakdown interpretation using band diagrams

(2) At collision, electron produces new electron–hole (e-h) pairs
Avalanche breakdown interpretation using band diagrams

(3) These secondary e-h pairs in also acquire high energy and may in turn produce more e-h pairs.
Avalanche breakdown interpretation using band diagrams

(4) New electron hole pairs will add up to the reverse current
(5) New electron-hole pairs will also create more electron hole pairs (avalanche multiplication)
Breakdown voltage of the p-n junction

For a p⁺ - n junction
(Nₐ >> Nₖ):

\[ V_{abd} = \frac{\varepsilon_s F_{bd}^2}{2qN_d} \]
Si p-n junction. Example 1:
Maximum Field vs. Reverse Voltage

\[ F_m = \sqrt{\frac{2qN'(V_{bi} - V_r)}{\varepsilon \varepsilon_0}} \]

\( N = 1E15 \text{ cm}^3 \)
Si p-n junction. Example 2
Maximum Field vs. Doping

\[ F_m = \sqrt{\frac{2 q N'(V_{bi} - V_r)}{\varepsilon \varepsilon_0}} \]
Si p-n junction. Example 3
Maximum Voltage vs. Doping

\[
V_{\text{max}} = \frac{\varepsilon \varepsilon_0 F_{BD}^2}{2q N'}
\]

For Silicon, 
\[F_{BD} \approx 3 \times 10^5 \text{ V/cm}\]
Zener (tunneling) breakdown

If the junction is highly doped then the depletion region can be very thin:

\[
W = \sqrt{\frac{2\varepsilon \varepsilon_0 (V_{bi} - V_r)}{qN'}}
\]

At high reverse bias, the conduction and valence bands on the opposite sides of the junction are very close to each other.
Zener (tunneling breakdown)

At certain reverse bias, the electrons from the valence band of p-material can tunnel into the conductance band of n-material. The reverse current increases abruptly at a certain voltage:

\[ V_Z \approx \frac{4 E_G}{q} \]
Avalanche vs. Zener Breakdown comparison

Avalanche (100V – 5 kV)

Zener (5V – 10V)