Three-phase systems
Power stations generate so-called three phase voltages

A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator. Each winding comprises one phase of the generator. The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover, such as a steam or gas turbine. Rotation of the electromagnet induces a sinusoidal voltage in each winding. The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120°.
Three phase generator and waveforms

Three-phase generator is equivalent to three AC voltage sources synchronously generating three voltages with the same frequency $f$ and different phases $\Theta$ (shifted by $120^\circ$ each)
Y-connected three-phase voltage sources

Equivalent circuit of Y-connected ideal three-phase voltage sources

Equivalent circuit of Y-connected three-phase voltage sources with winding impedances
Δ-connected three-phase voltage sources

Equivalent circuit of Δ-connected ideal three-phase voltage sources

Equivalent circuit of Δ-connected three-phase voltage sources with winding impedances
Advantages and importance of three-phase systems

1. The instantaneous power in three-phase system can be constant (not pulsating as in a single phase circuit).
2. For the same transmitted power, three-phase systems consume less wires.
3. For the above two reasons, most of the power distribution systems in US and over the world, use three-phase systems.
Y-Y three-phase system
Y-Y three-phase system

Let us find the voltage of the node N. This is a single-node circuit.
Assume:
\[ Z_{ga} = Z_{gb} = Z_{gc} \]
\[ Z_{1a} = Z_{1b} = Z_{1c} \]
\[ Z_A = Z_B = Z_C \]

The circuits that meet these conditions, are called balanced three-phase circuits.

Let us define the total impedances:
\[ Z_{AT} = Z_{ga} + Z_{1a} + Z_A \]
\[ Z_{BT} = Z_{gb} + Z_{1b} + Z_B \]
\[ Z_{CT} = Z_{gc} + Z_{1c} + Z_C \]

Y-matrix for the node N:
\[ Y = 1/Z_{AT} + 1/Z_{BT} + 1/Z_{CT} + 1/Z_0 = 3/Z_{AT} + 1/Z_0 \]

Current source matrix:
\[ I_s = V_a/Z_{AT} + V_b/Z_{BT} + V_c/Z_{CT} = (V_a + V_b + V_c)/Z_{AT} \]

Node potential:
\[ V_N = Y^{-1} I_s = Z_{AT}Z_0/(3Z_0+Z_{AT}) * (V_a + V_b + V_c) / Z_{AT} = Z_0/(3Z_0+Z_{AT}) * (V_a + V_b + V_c) \]
Y-Y three-phase system (cont.)

Node potential: \( V_N = \frac{Z_0}{(3Z_0 + Z_{AT})} \cdot (V_a + V_b + V_c) \)

Consider the sum of three phase voltages: \( (V_a + V_b + V_c) \)

Assuming equal amplitudes and 120° phase shifts:

Phase a: \( V_a \angle 0 \)
Phase b: \( V_b = V_a \angle 120^0 \)
Phase c: \( V_c = V_a \angle -120^0 \)

Using Euler’s formula for the complex amplitudes \( V_b \) and \( V_c \):
\[
V_b = V_a [\cos(120^0) + i \sin(120^0)]
\]
\[
V_c = V_a [\cos(120^0) - i \sin(120^0)]
\]
\[
V_b + V_c = V_a [\cos(120^0) + i \sin(120^0)] + V_a [\cos(120^0) - i \sin(120^0)] = V_a [2\cos(120^0)]
\]

Note that \( \cos(120^0) = -0.5 \);
Hence, \( V_b + V_c = V_a \cdot 2 \cdot (-0.5) = -V_a \);
Hence,

\[
(V_a + V_b + V_c) = 0 \quad \text{and} \quad V_N = 0
\]
Voltages and currents in Y-Y three-phase system

\[ V_N = 0: \]

in balanced three-phase circuits, the voltage across the neutral wire equals zero

In principal, the neutral wire can be eliminated or at least made less conducting.

Load currents: \( I_A = (V_a - V_N) / Z_T = V_a / Z_T \), as in a single phase circuit.
Similarly, \( I_B = V_b / Z_T \)
\( I_C = V_c / Z_T \)

However, unlike a single-phase circuit, the current through the neutral wire \( N \)

\( I_N = 0 \) as the voltage \( V_N = 0 \)

Using the KCL, we conclude that \( I_A + I_B + I_C = 0 \)
Instantaneous Load Powers in Y-Y system

Assuming equal amplitudes \( V_M \) and \( 120^\circ \) phase shifts:

\[
\begin{align*}
v_a &= V_M \cos(\omega t) \\
v_b &= V_M \cos(\omega t + 120^\circ) \\
v_c &= V_M \cos(\omega t - 120^\circ)
\end{align*}
\]

In balanced circuit, the load currents have the same amplitude and same phase shift with respect to the corresponding phase voltages:

\[
\begin{align*}
i_a &= I_M \cos(\omega t - \Theta) \\
i_b &= I_M \cos(\omega t - \Theta + 120^\circ) \\
i_c &= I_M \cos(\omega t - \Theta - 120^\circ)
\end{align*}
\]

Instantaneous powers: \( p(t) = i(t) \cdot v(t) \):

\[
\begin{align*}
p_a &= I_M V_M \cos(\omega t) \cos(\omega t - \Theta) \\
p_b &= I_M V_M \cos(\omega t + 120^\circ) \cos(\omega t - \Theta + 120^\circ) \\
p_c &= I_M V_M \cos(\omega t - 120^\circ) \cos(\omega t - \Theta - 120^\circ)
\end{align*}
\]
Instantaneous Load Powers in Y-Y system

Total instantaneous power: $p_T = p_a + p_b + p_c$:

$$p_T = I_M V_M \left[ \cos(\omega t) \cos(\omega t - \Theta) + \cos(\omega t + 120^\circ) \cos(\omega t - \Theta + 120^\circ) + \cos(\omega t - 120^\circ) \cos(\omega t - \Theta - 120^\circ) \right]$$

Applying the trigonometric identity:

$$\cos a \cos b = \frac{1}{2} \cos (a + b) + \frac{1}{2} \cos (a - b)$$

$$\cos(\omega t) \cos(\omega t - \Theta) = \frac{1}{2} [\cos(2\omega t - \Theta) + \cos(\Theta)]$$

$$\cos(\omega t + 120^\circ) \cos(\omega t - \Theta + 120^\circ) = \frac{1}{2} [\cos(2\omega t - \Theta + 240^\circ) + \cos(\Theta)]$$

$$\cos(\omega t - 120^\circ) \cos(\omega t - \Theta - 120^\circ) = \frac{1}{2} [\cos(2\omega t - \Theta - 240^\circ) + \cos(\Theta)]$$

Applying the trigonometric identity:

$$\cos a + \cos b = 2 \cos \frac{1}{2} (a + b) \cos \frac{1}{2} (a - b)$$

$$\frac{1}{2} [\cos(2\omega t - \Theta + 240^\circ) + \cos(2\omega t - \Theta - 240^\circ)] = \frac{1}{2} * 2 \cos(2\omega t - \Theta) \cos(240^\circ) = - \frac{1}{2} \cos(2\omega t - \Theta); \text{ Note, } \cos(240^\circ) = -1/2;$$

The total power $p_T$ is now given by:

$$p_T = \frac{1}{2} V_M I_M [3 \cos(\Theta)]; \text{ The total power does not have time dependence.}$$

Using effective voltages and currents: $V_{\text{rms}} = V_M / \sqrt{2}$ and $I_{\text{rms}} = I_M / \sqrt{2}$

$$p_T = V_{\text{rms}} I_{\text{rms}} 3 \cos(\Theta); \text{ The total power equals to the sum of the phase average powers}$$
Complex Powers in Y-Y system

\[ P_A = V_{\text{Arms}} I_{\text{Arms}} \cos(\theta_A), \]

\[ P_B = V_{\text{Brms}} I_{\text{Brms}} \cos(\theta_B), \]

\[ P_A = V_{\text{Crms}} I_{\text{Crms}} \cos(\theta_C), \]

\( \theta \) is the phase shift between the voltage and the current.

In balanced circuits, all the amplitudes and phase shifts are equal, hence:

\[ P_T = 3 \ V_{\text{rms}} I_{\text{rms}} \cos(\theta) \]
Delta to Y transformation

See ch. 11 for more details on Delta – Y connections in three-phase systems

\[
Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c},
\]

\[
Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c},
\]

\[
Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}.
\]

\[
Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1},
\]

\[
Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2},
\]

\[
Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}.
\]