Capacitors

The voltage $V$ between the plates and the charge $Q$ accumulated on each of the plates are related:

$$Q = C \times V$$

Where $C$ is a **capacitance**;

The capacitance units are Farads (F) \[1F = \frac{1C}{1V}\];

1 Farad (F) is the capacitance of a capacitor that accumulates the charge of 1 Coulomb (C) when the voltage between its plates is 1 Volt (V)
Electric current through capacitor

When the voltage changes, the current flows “through” a capacitor:

\[ Q = C \times V \]

\[ I = \frac{\partial Q}{\partial t} = C \frac{\partial V}{\partial t} \]

There is no actual electron drift through the gap between the plates. However, the charges on one plate induce the charges on the other plate so that there the current flows in the circuit.
Inductors

The inductor is typically a coil of wire (a solenoid). The inductance is the ability of inductor to resisting any change of electric current through the coil.

The voltage drop across the inductor in response to the current change:

\[ V = L \frac{\partial I}{\partial t} \]

The coefficient L is called “inductance”
The circuit symbol for inductor

The circuit diagram symbol for inductance

The unit for inductance is Henry

\[ V = L \frac{\partial I}{\partial t} \]

\[ L = V \left/ \left( \frac{\partial I}{\partial t} \right) \right. \]

1 H is the inductance of the coil that generates the voltage of 1V when the current change rate is 1 A per 1 s: \( 1H = 1V \times 1s / 1A \)
AC circuit components

Resistor

\[ v = R \times i \]

Capacitor

\[ i = C \times \frac{dv}{dt} \]

Inductor

\[ v = L \frac{di}{dt} \]

Note: we use low case letters for the time dependent variables and UPPER CASE letters for the AMPLITUDES.
Resistors in AC circuits

Let the source voltage, \( v(t) = V_M \cos (\omega t + \Theta) \)

Resistor current is given by the Ohm’s law:

\[
i_R = \frac{v_R}{R} = \frac{V_M}{R} \cos (\omega t + \Theta)
\]

Note:

- the current has the same type of the waveform;
- the angular frequency \( \omega \) does not change;
- the phase angle does not change
- the current amplitude \( I_M = \frac{V_M}{R} \) (as in DC circuits)
Capacitors in AC circuits

Let the source voltage, \( v(t) = V_M \cos (\omega t + \Theta) \)

Capacitor current:

\[
i_C = C \times \frac{d}{dt} v = C \times \frac{d}{dt} \left( V_M \cos (\omega t + \Theta) \right) = C \times \omega \times V_M \sin (\omega t + \Theta) = C \times \omega \times V_M \cos (\omega t + \Theta + \pi / 2)
\]

Note:

- the current has the same type of the waveform (sinusoidal);
- the angular frequency \( \omega \) does not change;
- the phase angle increases by \( \pi / 2 \);
- the current amplitude \( I_M = V_M \times (\omega \times C) \)
Inductors in AC circuits

Let the source current, \( i(t) = I_M \cos(\omega t + \Theta) \)

Inductor current

\[
v = L \frac{di}{dt}
\]

\[
v = L \times \frac{di}{dt} = - L \times \omega \times I_M \sin(\omega t + \Theta) =
\]

\[
= L \times \omega \times I_M \cos(\omega t + \Theta + \pi/2)
\]

Note:

• the current has the same type of the waveform;
• the angular frequency \( \omega \) does not change;
• the phase angle increases by \( \pi/2 \)
• the voltage amplitude \( V_M = I_M \times (\omega \times L) \)
R, L and C components in AC circuits: summary

• The currents through and the voltages across R, L or C components in AC circuits have sinusoidal waveforms.
• The current and voltage angular frequency $\omega$ (and hence, the frequency $f$ and the period $T$) are the same.
• There is phase difference between currents and voltages for C and L.

• Hence, to fully characterize the current – voltage relationships in AC circuits, only the amplitudes and phase angles are to be found.
Geometrical representation of amplitudes and phase angles

For any given sinusoidal voltage \( v(t) \) with the angular frequency \( \omega \), we need to know the amplitude and the phase angle to fully describe it.

\[ v(t) = V_M \sin (\omega t + \phi) \]

Uniquely and completely describes this AC voltage (assuming the angular frequency \( \omega \) is known)

The arrow of the length \( V_M \) making an angle \( \phi \) with the horizontal axis (x), also called vector, or phasor \( V_M \perp \phi \)
Geometrical representation of amplitudes and phase angles

Note: according to the sign convention, *positive* angles rotate the phasors *counterclockwise*.
Example

in an AC circuit with the angular frequency $\omega = 360 \text{ s}^{-1}$, the source voltage vector is $V_{BM} = 10 \text{ V} \angle 0^\circ$ (zero angle), the vector of the current is $I_M = 7 \text{ mA} \angle \pi/4$

What are the actual voltage and current in the circuit?

The source voltage:

$$v_B(t) = V_{BM} \sin (\omega t) = 10 \sin (360 \times t) \text{ V}$$

The current:

$$i_C(t) = I_M \sin (\omega t + \pi/4) = 7 \sin (360 \times t + \pi/4) \text{ mA}$$