Solving Networks with Multiple Sources
Using Superposition Method

Consider the circuit solved earlier using a nodal analysis.
The solution was: $\phi_1 = 9.09 \text{ V}; \phi_2 = 10.91 \text{ V}$

The superposition technique solves the circuit by TURNING ON ONE source at a time and ZEROING all the others.

Zeroing a voltage source

If the voltage of the voltage source $V_s = 0$, then the voltage $V_{AB} = 0$.
The potentials of the point A and point B are equal.
Compare to the wire connecting points A and B:
No matter what current is flowing through the wire, $V_{AB} = 0$.
The voltage source with ZERO voltage is equivalent to a wire (short-circuit) connecting its terminals.

Zeroing a current source source

If the current of the current source $I_s = 0$, then the current between the point A and point B is zero.
No matter what the voltage $V_{AB}$ is, the current between A and B is zero.
The current source with ZERO current is equivalent to a break (an open circuit) between its terminals.

Step 1. Consider the circuit with the current source $I_s$ zeroed

The circuit can be solved by series – parallel technique.

$R_{24} = R_2 + \frac{R_4}{R_3 + R_4} = 12 \text{ Ohm}$
$R_{324} = R_3 \frac{R_{24}}{R_3 + R_{24}} = \frac{5 \times 12}{17} = 3.53 \text{ Ohm}$
$R_{eq1} = R_1 + R_{324} = 1 + 3.53 = 4.53 \text{ Ohm}$

Total partial current $I_{11} = \frac{V_s}{R_{eq1}} = \frac{10}{4.53} = 2.2 \text{ A}$
The partial potential $\phi_{11} = V_{s1} - I_{11} \times R_1 = 10 \text{ V} - 2.2 \times 1 \text{ Ohm} = 7.8 \text{ V}$

Step 2. Consider the circuit solved earlier using a nodal analysis.
The solution was: $\phi_1 = 9.09 \text{ V}; \phi_2 = 10.91 \text{ V}$
Step 2. Consider the circuit with the voltage source $V_s$ zeroed

The circuit can be solved by series – parallel technique.
$R_{13} = R_1 * R_3 / (R_1 + R_3) = 1\,\text{Ohm} * 5/6 = 0.83\,\text{Ohm}$
$R_{213} = R_2 + R_{13} = 2 + 0.83 = 2.83\,\text{Ohm}$
$R_{eq2} = R_4 * R_{213} / (R_4 + R_{213}) = 10 * 2.83 / 12.83 = 2.2\,\text{Ohm}$
Total partial current $I_{T2} = I_{S2}$
The partial potential $\phi_{22} = I_{S2} * R_{eq2} = 2\,\text{A} * 2.2\,\text{Ohm} = 4.4\,\text{V}$;

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**Linearity Property**

For any circuit, if the voltage (or current) of one the voltage (or current) source changes $k$ times, all the currents and all the voltages in the circuit due to this source change $k$-times.

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**Example 1**

Use linearity to find the actual value of $I_o$ in the circuit shown below.

Assume $I_o = 1\,\text{A}$;
$V_1 = I_o * (3\,\text{Ohm} + 5\,\text{Ohm}) = 8\,\text{V}$;
$\Rightarrow I_1 = 8\,\text{V} / 4\,\text{Ohm} = 2\,\text{A}$
$\Rightarrow I_2 = I_1 + I_o = 3\,\text{A}$;
$\Rightarrow V_2 - V_1 = I_o * 2\,\text{Ohm} = 6\,\text{V}$; $\Rightarrow V_2 = V_1 + 6\,\text{V} = 14\,\text{V}$
$\Rightarrow I_3 = V_2 / 7\,\text{Ohm} = 14/7 = 2\,\text{A}$
$\Rightarrow I_4 = I_3 + I_o = 3\,\text{A} + 2\,\text{A} = 5\,\text{A}$
The actual current $I_o = I_3 = 15\,\text{A}$, is 3 times higher.
Hence the assumed $I_o$ value needs to be corrected by a factor of 3, hence the actual value is $I_o = 3\,\text{A}$
Example 2
Solve the circuit using source transformation

Consider the circuit solved earlier using a nodal analysis. The solution was:
\[ \phi_1 = 9.09 \text{ V}; \quad \phi_2 = 10.91 \text{ V} \]

Transform the current source into a voltage source
Equivalent Thevenin voltage, \( V_{TH} = V_{OC} = 2 \text{ A} \times 10 \text{ Ohm} = 20 \text{ V} \).
Equivalent series resistance is the resistance at the current source terminals with the current source set to zero, \( R_{TH} = 10 \text{ Ohm} \).

Modified circuit after current source transformation
His circuit has a single unknown potential \( \phi_1 \), for which the KCL equation is:
\[ \phi_1 \left( 1/R_1 + 1/R_3 + 1/R_{TH} \right) = V_{ST}/R_1 + V_{TH}/R_{TH} \]
Where \( R_{TH} = R_2 + R_{TH} = 12 \text{ Ohm} \)
\[ \phi_1 * 1.28 = 11.67; \quad \phi_1 = 9.09 \text{ V} \]

Example 3
Replace the circuit attached to a-b with the voltage source
First, find the open-circuit voltage between terminals a and b:
\( V_{TH} = V_{OC}(\text{open-circuit}) \)
Applying nodal analysis to the node “1”:
\[ G1 = 1/5 + 1/20 = 0.25; \] (there is no current through the 4-Ohm resistor when the circuit is open);
\[ I_s = 25/5 + 3 = 8 \text{ A} \]
\[ V_{loc} = G(-1)*I_s = (1/0.25)*8 = 32 \text{ V} \]
\[ V_{TH} = V_{oc}(\text{open circuit}) = 32 \text{ V} \]
Example (cont.)

Second, find the resistance between a and b terminals when all the sources are set to zero. 
\[ R_{ab} = \frac{5}{20} + 4 = 4 + 4 = 8 \text{ Ohm} \]

\[ R_{TH} = 8 \text{ Ohm} \]

Example (cont.)

The circuit on the left can be replaced with the Thevenin equivalent voltage source having the equivalent open-circuit voltage \( V_{TH} = 32 \text{ V} \) and equivalent resistance \( R_{TH} = 8 \text{ Ohm} \).

If the load resistance \( R_L \) connected to the terminals a and b needs to be optimized for the maximum power dissipated in it, then \( R_{opt} = R_{TH} \).

In the above circuit, the optimal load resistance, \( R_{opt} = 8 \text{ Ohm} \).

Example 4:

**convert voltage source into current source**

\[ V_S = 9 \text{ V} \]

1) find the output resistance with the voltage source zeroed: 
\[ R_{out} = R_S = 3 \text{ \Omega} \]

2) find the short-circuit current between the terminals: 
\[ I_{SC} = \frac{V_S}{R_S} = \frac{9 \text{ V}}{3 \text{ \Omega}} = 3 \text{ A} \]

Equivalent (Norton) current source parameters:
\[ I_N = I_{SC} = \frac{V_S}{R_S} = 3 \text{ A} \]
\[ R_N = R_{OUT} = R_S = 3 \text{ \Omega} \]