Ideal voltage source grounded, connected to multiple resistors

Although the voltage source $V_{B1}$ is connected to the nodes "1" and "3" through different resistors, the same rules apply:

$G_{11} = (1/R_1+1/R_2+1/R_3)$ - sum of all the conductances connected directly to node 1

$G_{22} = (1/R_3+1/R_4+1/R_5)$ - sum of all the conductances connected directly to node 2

$G_{33} = (1/R_5+1/R_6)$ - sum of all the conductances connected directly to node 3

$G_{12} = G_{21} = -1/R_3$ - the conductance between nodes 1 and 2 taken with negative sign;

$G_{13} = G_{31} = 0$; $G_{23} = G_{32} = -1/R_5$;

$I_{S1} = V_{B1}/R_1$; $I_{S2} = 0$; $I_{S3} = V_{B1}/R_6 + I_{S2}$; - current sources connected to nodes 1,2,3
The circuit contains a voltage source $V_{B1}$ connected between two nodes.

The problem of using conventional nodal analysis is that the current through $V_{B1}$ cannot be expressed by any formula (like an Ohm’s law for resistors). Therefore, we cannot write the KCL for the nodes ‘1” and “2”.

However, we can write the KCL for the combination of nodes “1” and “2” – the 
**supernode “1 – 2”:**  
\[-I_1 - I_2 + I_3 + I_5 = 0\]  
\[-\frac{\phi_1}{R_1} - \frac{\phi_2}{R_2} + \frac{\phi_3 - \phi_1}{R_5} + \frac{\phi_3 - \phi_2}{R_3} = 0\]  
\[(1)\]

For the node “3” we can use the KCL:  
\[\frac{\phi_1 - \phi_3}{R_5} + \frac{\phi_2 - \phi_3}{R_3} - \frac{\phi_3}{R_4} = 0\]  
\[(2)\]

We now have 3 unknown variables: $\phi_1$, $\phi_2$, $\phi_3$ and two equations for them (1, 2)

The third equation comes from the known voltage of the voltage source:

\[\phi_1 - \phi_2 = V_{B1}\]  
\[(3)\]

This system of three equations can be solved using regular linear algebra methods
We cannot formally apply the KVL to the mesh #2, because we do not know the voltage across the current source. Note that the mesh current $i_2$ is already known because $i_2 = I_{s2}$, so can we simply drop the KCL equation for the mesh #2 and only use the one for the mesh #1:

$$V_{s1} - i_1 R_1 - (i_1 - i_2) R_3 = 0$$

$$i_2 = I_{s2}$$
Current source shared between two meshes

\[ V_{S1} = 20 \text{ V} \]

\[ I_{S2} = 6 \text{ A} \]

\[ R_1 = 6 \Omega \]
\[ R_2 = 2 \Omega \]
\[ R_3 = 10 \Omega \]
\[ R_4 = 4 \Omega \]
Current source shared between two meshes

\[ V_{S1} - i_1 R_1 - i_2 R_3 - i_2 R_4 = 0 \]

\[ -i_1 + i_2 = I_{S2} \]