Building the G matrix:

\[ G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}; \] - sum of all the conductances connected to node 1

\[ G_{12} = -\frac{1}{R_2}; \] the conductance between nodes 1 and 2 taken with negative sign;

\[ G_{21} = G_{12}; \]

\[ G_{22} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}; \] - sum of all the conductances connected to node 2

The circuit has two unknown potentials.

Therefore, in the nodal equation:

\[ [\phi] \times [G] = [I_s] \]

\([G]\) is 2 x 2 matrix:

\[ [G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \]

and \([I_s]\) is 2 x 1 matrix:

\[ [I_s] = \begin{bmatrix} I_{s1} \\ I_{s2} \end{bmatrix} \]
First, let us build the G and $I_S$ matrices using the KCL. We obtain:

\[
\begin{bmatrix}
\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\
-\frac{1}{R_2} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}
\end{bmatrix}
\begin{bmatrix}
\varphi_1 \\
\varphi_2
\end{bmatrix}
= \begin{bmatrix}
\frac{V_{B1}}{R_1} \\
\frac{I_{S2}}{}
\end{bmatrix}
\]

The circuit has two unknown potentials. Therefore, in the nodal equation:

\[
[\varphi] \times [G] = [I_S]
\]

$[G]$ is 2 x 2 matrix:

\[
[G] = \begin{bmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{bmatrix}
\]

and $[I_S]$ is 2 x 1 matrix:

\[
[I_S] = \begin{bmatrix}
I_{S1} \\
I_{S2}
\end{bmatrix}
\]
Building the $I_S$ matrix:

$I_{S1} = V_{B1} / R_1 =$ equivalent external source current flowing into node 1;

$I_{S2}$ - external source current flowing into node 2;
Building nodal matrices by inspection: general rules

For ANY NETWORK with \( n \) unknown potentials:

\[
\begin{bmatrix}
\varphi
\end{bmatrix} \times \begin{bmatrix} [G] \end{bmatrix} = \begin{bmatrix} [I_S] \end{bmatrix}
\]

and the solution is:

\[
\begin{bmatrix}
\varphi
\end{bmatrix} = \begin{bmatrix} [G]^{-1} \end{bmatrix} \times \begin{bmatrix} [I_S] \end{bmatrix}
\]

\([G]\) is the conductance matrix \([n \times n]\) in size,

\([I_S]\) is the source currents matrix \([n \times 1]\) in size

(1) Diagonal elements \( G_{jj} \) (gray strip):

\( G_{KK} = \) sum of all the node conductances connected to node “K” \((K=1,2, \ldots n)\)

(2) Off-diagonal elements \( G_{KL} \) (blue triangles):

\( G_{KL} = - \) (conductance between nodes K and L), i.e. between 1 and 2, 1 and 3, \ldots n3 and nn.

Note that \( G_{KL} = G_{LK} \);
Building nodal matrices by inspection: general rules (cont.)

For ANY NETWORK with \( n \) unknown potentials:

Nodal equation: \([\phi] \times [G] = [I_S]\) and the solution is: \([\phi] = [G]^{-1} \times [I_S]\)

\([G]\) is the conductance matrix \([n \times n]\) in size,
\([I_S]\) is the source currents matrix \([1 \times n]\) in size

(3) External sources \( I_K \)
\( I_K = \) sum of all source currents flowing into the node \( K \)

Currents from currents source are simply equal to the current source value;
the currents from voltage sources are calculated by dividing the source voltage by
the resistor connected between the source and the node.
Example

Build the nodal matrices and find all the node potentials:
MATLAB code:

clear all
G=zeros(3,3);
Is=zeros(3,1);
G(1,1)=1/46+1/72+1/23;
G(1,2)=-1/23;
G(2,1)=G(1,2);
G(1,3)=0;
G(3,1)=0;
G(2,2)=1/23+1/4+1/37;
G(2,3)=-1/37;
G(3,2)=-1/37;
G(3,3)=1/37+1/56+1/49;
Is(1,1)=1/46;
Is(2,1)=1/4;
Is(3,1)=1.5/56;
Fi=G^(-1)*Is;
Fi

MATLAB results:

φ₁ = 0.8 V
φ₂ = 0.96 V
φ₃ = 0.81 V

What is the current through R₅? Iₐ₅ = (φ₂ - V₂)/R₅ = (0.96 - 1)/4 = -0.05 A;
Negative sign means that the current flows “downward”.
Special cases of nodal matrices: ideal voltage source grounded

Although the voltage source $V_{B1}$ is connected to the nodes ”1” and “3” through different resistors, the same rules apply:

$G_{11} = (1/R_1+1/R_2+1/R_3)$ - sum of all the conductances connected directly to node 1

$G_{22} = (1/R_3+1/R_4+1/R_5)$ - sum of all the conductances connected directly to node 2

$G_{33} = (1/R_5+1/R_6)$ - sum of all the conductances connected directly to node 3

$G_{12} = G_{21} = -1/R_3$ - the conductance between nodes 1 and 2 taken with negative sign;

$G_{13} = G_{31} = 0; G_{23} = G_{32} = -1/R_5$;

$I_{S1} = V_{B1}/R_1; I_{S2} = 0; I_{S3} = V_{B1}/R_6 + I_{S2}$; - current sources connected to nodes 1,2,3
Special cases of nodal matrices: ideal voltage source non-grounded

The circuit contains a voltage source $V_{B1}$ connected between two nodes.

The problem of using conventional nodal analysis is that the current through $V_{B1}$ cannot be expressed by any formula (like an Ohm’s law for resistors). Therefore, we cannot write the KCL for the nodes ‘1” and “2”.

However, we can write the KCL for the combination of nodes “1” and “2” – the supernode “1 – 2”: $-I_1 - I_2 + I_3 + I_5 = 0$, or: 

$$\frac{-\varphi_1}{R_1} - \frac{\varphi_2}{R_2} + \frac{\varphi_3 - \varphi_1}{R_5} + \frac{\varphi_3 - \varphi_2}{R_3} = 0 \quad (1)$$

For the node “3” we can use the KCL:

$$\frac{\varphi_1 - \varphi_3}{R_5} + \frac{\varphi_2 - \varphi_3}{R_3} - \frac{\varphi_3}{R_4} = 0 \quad (2)$$

We now have 3 unknown variables: $\varphi_1, \varphi_2, \varphi_3$ and two equations for them (1, 2)

The third equation comes from the known voltage of the voltage source:

$$\varphi_1 - \varphi_2 = V_{B1} \quad (3)$$

This system of three equations can be solved using regular linear algebra methods.