Inductors

The inductor is typically a coil of wire (a solenoid). The inductance is the ability of inductor to resisting any change of electric current through the coil.

The voltage drop across the inductor in response to the current change:

\[ V = L \frac{\partial I}{\partial t} \]

The coefficient L is called “inductance”
The circuit symbol for inductor

\[ L \]

The circuit diagram symbol for inductance

The unit for inductance is Henry

\[ V = L \frac{\partial I}{\partial t} \]

\[ L = \frac{\sqrt{\frac{V}{\partial I / \partial t}}} \]

1 H is the inductance of the coil that generates the voltage of 1V when the current change rate is 1 A per 1 s: \[ 1H = 1V \times 1s / 1A \]
Factors affecting the Inductance

Greater number of turns \( N \), results in greater inductance (stronger magnetic field)

\[ L = \frac{N^2 \mu_0 A}{l} \]

\( \mu_0 \) is the magnetic permittivity of the vacuum, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)

Greater coil area \( A \) results in greater inductance (stronger magnetic field)
Factors affecting the Inductance

Certain materials increase the magnetic field.
This increase is described by a factor called magnetic permittivity $\mu$

The inductance of a coil having $N$ turns, the area $A$, the length $l$ and the magnetic permittivity $\mu$ of the filling material:

$$L = \frac{N^2 \mu \mu_0 A}{l}$$
Energy stored in inductor

\[ W_L = \frac{L \times I^2}{2} \]
Parallel connection of inductors

\[ V = V_B \]

\[
\begin{align*}
V &= L_1 \frac{\partial I_1}{\partial t} \\
V &= L_2 \frac{\partial I_2}{\partial t}
\end{align*}
\]

The voltage \( V_1 = V_2 \); The total current \( I_T = I_1 + I_2 \).

\[
\begin{align*}
\frac{V}{L_1} &= \frac{\partial I_1}{\partial t} \\
\frac{V}{L_2} &= \frac{\partial I_2}{\partial t} \\
\frac{V}{L_1} + \frac{V}{L_2} &= \frac{\partial I_1}{\partial t} + \frac{\partial I_2}{\partial t} \\
V \left( \frac{1}{L_1} + \frac{1}{L_2} \right) &= \frac{\partial (I_1 + I_2)}{\partial t} = \frac{\partial I_T}{\partial t}
\end{align*}
\]

\[
V = \left[ 1 \left( \frac{1}{L_1} + \frac{1}{L_2} \right) \right] \frac{\partial I_T}{\partial t};
\]

\[
1/L_{\text{par}} = 1/L_1 + 1/L_2; \quad L_{\text{par}} = \frac{L_1 L_2}{L_1 + L_2}
\]
Series connection of inductors

Inductors $L_1$ and $L_2$ are connected in series:
- the current $I$ through $L_1$ and $L_2$ is the same;
- the total voltage drop $V_T = V_1 + V_2$.

From the KVL, $V_B = V_{Tot} = V_1 + V_2$.

$$V_{Tot} = \left( L_1 \frac{\partial I}{\partial t} + L_2 \frac{\partial I}{\partial t} \right) = (L_1 + L_2) \frac{\partial I}{\partial t} = L_s \frac{\partial I}{\partial t};$$

The equivalent series inductance,

$$L_s = L_1 + L_2$$
Transients in Inductors

\[ V_L = L \frac{\partial I}{\partial t} \]

When the switch is turned on, \( dI/dt > 0 \), hence \( V_L > 0 \). The inductor acts a dynamic (transient) resistance.

The current through inductor cannot change instantly:

Initial condition: \( I_L(0+) = I_L(0-) = 0 \)
Transients in Inductors

When the switch is turned off, $dI/dt < 0$, hence $V_L < 0$. The inductor acts as a dynamic current source (trying to maintain the same current as before switching).

*The current through inductor cannot change instantly:*

Initial condition: $I_L(0+) = I_L(0-) = V_B/R$
Transient current in R-L circuit

\[ i(t) = I_{DC} \times \left( 1 - e^{-Rt/L} \right) \];

where \( I_{DC} = V_B / R \);

\[ v_L(t) = V_B \ e^{-Rt/L} \];

Characteristic time of the RL circuit:

\[ \tau_{RL} = L/R \]
General form of step response of R-L circuit

\[ i_L = i_{LF} + (i_{L0} - i_{LF}) e^{-t/\tau} \]

\( \tau = L/R \)

\( i_{L0} \) is the inductor current right before the commutation;
\( i_{LF} \) is the inductor current after all the transient processes are over.

R is the total resistance connected to the inductor after commutation (all the sources are zeroed to find the equivalent total resistance)
General form of step response of R-C and R-L circuits

**R-C circuit**

\[ V_s \]

\[ \begin{array}{c}
\text{R} \\
\text{C} \\
\end{array} \]

\[ x = x_F + (x_0 - x_F) e^{-t/\tau} \]

RC-circuit: \( x = v_C \)

\[ v_C = v_{CF} + (v_{C0} - v_{CF}) e^{-t/\tau} \]

\( \tau = RC \)

**R-L circuit**

\[ V_s \]

\[ \begin{array}{c}
\text{L} \\
\text{R} \\
\end{array} \]

\[ x_0 = \text{initial value}; \quad x_F = \text{final value}; \]

RL-circuit: \( x = i_L \)

\[ i_L = i_{LF} + (i_{L0} - i_{LF}) e^{-t/\tau} \]

\( \tau = L/R \)
Example 1

Delayed alarm circuit

Sensor switch: \textbf{S1}.
Electronic switch \textbf{S2} triggers the alarm system when the voltage across it exceeds the preset threshold value \( V_T \).
\( R = 10\, \text{k} \); \( C = 0.5\, \text{mF} \); \( V_B = 4.5\, \text{V} \).
The required time delay between the switch \textbf{S1} turn-on and triggering switch \textbf{S2} must be \( t_t = 3\, \text{s} \).

\textbf{What threshold voltage} \( V_T \) \textbf{must the switch} \textbf{S2} \textbf{be tuned to?}
Example problem

The switch in the circuit shown in Fig. 7.28 has been open for a long time. At \( t = 0 \) the switch is closed.

**Find the expression for \( i \) at \( t>0 \)**

\[
i = i_F + (i_0 - i_F) e^{-t/\tau}
\]

\[
i_0 = \frac{20 \text{ V}}{(1 \Omega + 3 \Omega)} = 5 \text{ A} \quad i_F = \frac{20 \text{ V}}{1 \Omega} = 20 \text{ A}
\]
\[
\tau = \frac{L}{R} = \frac{80 \text{ mH}}{1 \Omega} = 80 \text{ ms}
\]

\[
i = 20 - 15e^{-t/0.08} \text{ A}
\]
The electronic switch alternates between open and closed approximately 10,000 times per second. (In some units, you can hear a high-pitched whistle resulting from incidental conversion of some of the energy to acoustic form.) While the electronic switch is closed, the battery causes the current in the inductor to build up. Then when the switch opens, the inductor forces current to flow through the diode, charging the capacitor. (Recall that the current in an inductor cannot change instantaneously.) Current can flow through the diode only in the direction of the arrow. Thus, the diode allows charge to flow into the capacitor when the electronic switch is open and prevents charge from flowing off the capacitor when the electronic switch is closed. Thus, the charge stored on the capacitor increases each time the electronic switch opens. Eventually, the voltage on the capacitor reaches several hundred volts. When the camera shutter is opened, another switch is closed, allowing the capacitor to discharge through the flash tube.