Maximum Power delivered to the load

Example: \( R_L = 100 \ \Omega \);
For ideal battery, the power in the load would be:

\[
P_L = \frac{V_B^2 R_L}{(R_i + R_L)^2}
\]

For real battery with the internal resistance \( R_i = 20 \ \Omega \),

\[
P_L = 9^2 V^2 \frac{100}{(100+20)^2} = 81*100/14400 = 0.56 \ W
\]
Maximum Power delivered to the load

\[ P_L = V_B^2 \frac{R_L}{(R_i + R_L)^2} \]

Note that, when \( R_L \ll R_i \), \( P_L \approx \frac{R_L}{R_i^2} \). Therefore, when \( R_L \), \( P_L \uparrow \).

When \( R_L \gg R_i \), \( P_L \approx \frac{1}{R_L} \), Therefore, when \( R_L \uparrow \), \( P_L \downarrow \).

Therefore, we expect that \( P_L(R_L) \) dependence has a maximum.
Maximum Power delivered to the load

\[ P_L = V_B^2 \frac{R_L}{(R_i + R_L)^2} \]

Let us normalize the \( P_L(R_L) \) formula:

\[ P_L = V_B^2 \frac{R_L}{R_i^2} \left( 1 + \frac{R_L}{R_i} \right)^2 = \frac{V_B^2}{R_i} \frac{R_L}{R_i} \left( 1 + \frac{R_L}{R_i} \right)^2 = P_i x \frac{1}{(1 + x)} \]

Where \( x = R_L/R_i \); \( P_i = V_B^2/R_i \)
Maximum Power delivered to the load

\[ P_L = V_B^2 \frac{R_L}{R_i^2} \left( 1 + \frac{R_L}{R_i} \right)^2 = \frac{V_B^2}{R_i} \frac{R_L}{R_i} \left( 1 + \frac{R_L}{R_i} \right)^2 = P_i \left( 1 + \frac{R_L}{R_i} \right) \]

Load matching condition: \( R_{L,\text{OPT}} = R_i \)

\[ P_{\text{MAX}} = \frac{P_i}{4} = \frac{V_B^2}{4R_i} \]
Series and parallel connection of voltage sources

**Series connection** is widely used to increase the output voltage of the voltage source.

The KVL for this circuit:

\[ V_{S1} - I*R_1 + V_{S2} - I*R_2 + V_{S3} - I*R_3 - I*R_L = 0 \]

\[ V_{S1} + V_{S2} + V_{S3} - I*(R_1 + R_2 + R_3) - I*R_L = 0 \]

Compare this with a single source circuit:

\[ V_S - I*R - I*R_L = 0 \]

The equivalent voltage,

\[ V_{SE} = V_{S1} + V_{S2} + V_{S3} \]

The equivalent internal resistance:

\[ R_{iE} = R_{i1} + R_{i2} + R_{i3} \]
Parallel connection of voltage sources is used to increase the maximum available current.

The KCL for this circuit:
\[ I_1 + I_2 + I_3 = I_L \]

Assuming all the sources to be identical:
\[ \frac{3V_S - 3\phi_L}{R_i} = \frac{\phi_L}{R_L}; \quad \Rightarrow \quad \frac{V_S - \phi_L}{(R_i/3)} = \frac{\phi_L}{R_L}; \]

\[ \phi_L \left( \frac{1}{R_L} + \frac{1}{R_i/3} \right) = \frac{V_S}{(R_i/3)}; \quad \rightarrow \quad V_L = \phi_L = V_S \frac{1}{(R_i/3)} \frac{R_L(R_i/3)}{R_L+(R_i/3)} = V_S \frac{R_L}{R_L+(R_i/3)} \]

Compare this \( V_L \) to the load voltage of a single source:
\[ V_L = V_S \frac{R_L}{R_i + R_L} \]

The equivalent voltage and internal resistance are:

\[ V_{SE} = V_S; \quad R_{iE} = \frac{R_i}{3} \]
Summary of the series and parallel voltage source connections

For the **series connection** of \( N \) identical voltage sources having the voltage \( V_S \) and the internal resistance \( R_i \) each, the equivalent voltage and internal resistance are:

\[
V_{SE} = N \times V_S; \\
R_{iE} = N \times R_i;
\]

For the **parallel connection** of \( N \) identical voltage sources having the voltage \( V_S \) and the internal resistance \( R_i \) each, the equivalent voltage and internal resistance are:

\[
V_{SE} = V_S; \\
R_{iE} = R_i / N;
\]
Current sources

An ideal current source is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

Current sources are mainly an abstraction as NO actual source can deliver any current unless connected to the finite load.

Example of practical current source: \( R_i >> R_L \) for any load resistance within a range.

The current in this circuit:

\[
I = \frac{V_B}{R_i + R_L};
\]

If \( R_i >> R_L \),

\[
I \approx \frac{V_B}{R_i};
\]
An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

An equivalent circuit of a **real current source**

The equivalent resistance:

$$R_{EQ} = \frac{R_i R_L}{R_i + R_L}$$

The voltage across the load

$$V_L = I_S \frac{R_i R_L}{R_i + R_L}$$

The load current

$$I_L = \frac{V_L}{R_L} = I_S \frac{R_i}{R_i + R_L}$$

The actual load current is smaller than the $I_S$. 
Current source example: Field-Effect Transistor

An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

Output current $I_L$ is almost independent on the output voltage (drain-to-source voltage)
Dependent Current and Voltage Sources

\[ v_s = \mu v_x \quad \text{(a)} \]
\[ i_s = \alpha v_x \quad \text{(b)} \]
\[ v_s = \rho i_x \quad \text{(c)} \]
\[ i_s = \beta i_x \quad \text{(d)} \]

The circuit symbols for:
(a) an ideal dependent voltage-controlled voltage source,
(b) an ideal dependent current-controlled voltage source,
(c) an ideal dependent voltage-controlled current source, and
(d) an ideal dependent current-controlled current source.
Voltage-controlled Current source example:
Field-Effect Transistor

Output current $I_L$ depends on the voltage applied to the gate of MOSFET