Electric forces exert on any charge placed in the electric field
The electric field $E$
has the magnitude and the direction (it is a vector)

The direction of the field is taken to be the direction of the force it
would exert on a positive test charge.

Given the electric field $E$, the force exerting on any charge $q$ can be
found as

$$F = E \times q$$
Potential energy in electric field

The electric force exerts on electric charge the same way as the gravitational force exerts on a body.

To lift the body up by the distance $h$, the work against the gravity force is:

$$ W = (\text{Force} \times \text{Distance}) = F \ h = mg \ h $$

To move the charge $q$ by distance $x$ against the field direction (up, in our example), the work against the electric force is

$$ W = (\text{Force} \times \text{Distance}) = F \ x = q \ E \ x $$
Potential energy in electric field (cont.)

The potential energy of a body in gravitational field:

\[ W_G = mgh \]

The potential energy of a charge in electric field:

\[ W_E = qE x \]

Electric field – gravitational field analogy

Charge \( q \) \( \quad \) Mass \( m \)

Electric field \( E \) \( \quad \) Gravitational acceleration \( g \)
Mechanical and Electrical potential energy - example

Potential energy of a charge in electric field:
\[ W_E = qE x \]

A positive charge of 1C was moved by 1 mm in the electric field of 10 N/C against the field lines. What is the change in the charge potential energy?

\[ q=1C; \quad x=1 \text{ mm}=10^{-3} \text{ m}; \quad E= 10 \text{ N/C} \]

\[ W_E = qE x = 1C \times 10 \text{ N/C} \times 10^{-3} \text{ m}= 10 \times 10^{-3} \text{ J}; \quad W_E \text{ increases} \]

Potential energy of a body in gravitational field:
\[ W_G = mgh \]

A body of the mass 1kg was lifted up by 1 mm in the gravitational field (\( g= 9.8 \text{ m/s}^2 \)). What is the change in the body potential energy?

\[ m=1\text{kg}; \quad h=1 \text{ mm}=10^{-3} \text{ m}; \quad g= 9.8 \text{ m/s}^2 \]

\[ W_G = mgh = 1\text{kg} \times 9.8 \text{ m/s}^2 \times 10^{-3} \text{ m}= 9.8 \times 10^{-3} \text{ J}; \quad W_G \text{ increases} \]
Potential energy in electric field (cont.)

A positive charge 1C moved from the point $x_1$ into point $x_2$ separated by 1 mm in the electric field of 10 N/C in the direction of field lines. What is the change in the charge potential energy?

Potential energy of a charge in electric field:
point $x_1$: $W_{E1} = qE x_1$; point $x_2$: $W_{E2} = qE x_2$;

$q=1C; \quad x=1 \text{ mm}=10^{-3} \text{ m}; \quad E=10 \text{ N/C}$

$\Delta W_E = W_{E2} - W_{E1}$

$\Delta W_E = qE (x_2 - x_1) = 1C \times 10 \times (-10^{-3}) = -10 \times 10^{-3} \text{ J}$;

Notes: 1) $W_E$ decreases – the electric field does the work
2) the absolute positions, $x_1$ and $x_2$ do not matter; only the difference $\Delta x = x_2 - x_1$
Electric potential

Potential is a potential energy of a unit charge in the electric field; it does not depend on the charge value:

$$\varphi = \frac{W_E}{q} \quad [\text{J/C}]$$

$$\varphi = E x \quad [(\text{N/C}) \times \text{m}] = [\text{J/C}]$$

Potential is always measured with respect to the reference (zero potential) level.

A positive charge of 1C was moved by 1mm in the electric field of 10 N/C against the field lines. What is the change in the charge potential energy?

$$q=1\text{C}; \Delta x=1 \text{ mm}=10^{-3} \text{ m}; \quad E=10 \text{ N/C}$$

$$W_E = qE \Delta x = 1\text{C} \times 10 \text{ N/C} \times 10^{-3} \text{ m}=10 \times 10^{-3} \text{ J}. \quad W_E \text{ increases}$$

A charge was lifted by 1mm from the reference plane in the electric field of 10 N/C against the field lines. What is the charge potential?

$$x=1 \text{ mm}=10^{-3} \text{ m}; \quad E=10 \text{ N/C}$$

$$\varphi = E x = 10 \text{ N/C} \times 10^{-3} \text{ m} = 10 \times 10^{-3} \text{ Nm/C}= 10 \times 10^{-3} \text{ J/C}$$
Electric potential definition

**Potential** is the potential energy of a unit charge in electric field:

$$\phi = \frac{W_E}{q}$$

Potential is measured in Volts (V)

1V is the potential that changes a potential energy of the unit charge of 1C (Coulomb) by 1 J (Joule)

$$1V = \frac{1\ J}{1C}$$

If the potential $\phi$ of any point in the electric field is known, the potential energy of any charge $Q$ can be found:

$$W_E = Q \times \phi$$
Electric field units: “V/m” and “N/C”

\[ \Phi = E \times ; \] from this

1 V = 1 N/C \times 1 m;

1 N/C = 1 V/m

\[ [E] = [N/C] = [V/m] \]

\textbf{V/m} is a commonly accepted unit for the electric field.
Summary of the electric force, field and potential concepts

• Electric forces exist in the space surrounding any charge.

• Electric forces exert on any charge located in the vicinity of the source charges.

• The magnitude of electric forces can be characterized by electric fields.
  Electric field is the electric force per unit charge.

• The potential energy of a charge in electric field is characterized by the potential.
  Potential (Volts) is the potential energy in electric field per unit charge.
Example problem 1

Negatively charged plate creates a uniform electric field of $10^3$ V/m.

(a) What is the potential of a point 1 mm above the plate?

\[ \phi = E \cdot x; \quad x = 1 \text{ mm} = 10^{-3} \text{ m} \]

\[ \phi = E \cdot x = 1.0 \times 10^3 \text{ V/m} \times 10^{-3} \text{ m} = 1.0 \text{ V}; \]

(b) What is the potential of a point that is 2 mm above the plate?

\[ \phi = 2.0 \text{ V}; \]

(c) What is the potential of a point directly on the plate?

\[ \phi = 0 \text{ V} \]
Example problem 2

Negatively charged plate creates a uniform electric field of $10^3 \text{ V/m}$.  
Point 1 is located 1 mm above the plate;  
point 2 is located 2 mm above the plate.  
What is the difference in the potentials of the two points?

$$\phi_1 = E x_1; \quad x = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\phi_1 = E x_1 = 1.0 \times 10^3 \text{ V/m} \times 10^{-3} \text{ m} = 1.0 \text{ V};$$

$$\phi_2 = E x_2; \quad x = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\phi_2 = E x_2 = 1.0 \times 10^3 \text{ V/m} \times 2 \times 10^{-3} \text{ m} = 2.0 \text{ V};$$

$$\phi_2 - \phi_1 = 2.0 \text{ V} - 1.0 \text{ V} = 1 \text{ V}$$
**Example problem 3**

Negatively charged plate creates a uniform electric field of $10^3$ V/m. What is the change in the potential of a charge that has been moved up by $d = 1$ mm?

The distance from each of the points 1 and 2 to the plate is unknown. Assume point 1 is $d_0$ mm away from the plate. For the point 2 the distance would be $(d_0 + d)$ as the charge moves UP.

\[ \phi_1 = E d_0; \quad (d_0 \text{ unknown}) \]
\[ \phi_2 = E (d_0 + d); \quad (d_0 \text{ unknown}) \]

The change in the potential

\[ \Delta \phi = \phi_2 - \phi_1 = E (d_0 + d) - Ed_0 = E d \]

\[ \Delta \phi = E d = 10^3 \text{ V/m} \times 10^{-3} \text{ mm} = 1 \text{ V} \]

*Important observation*: the potential difference *DOES NOT* depend on the absolute position of the starting point.
**Example problem 4**

What is the change in the potential energy of the charge $Q=1 \text{ nC}$ that has been moved from the point with the potential $2 \text{ V}$ to the point with the potential of $5 \text{ V}$?

The potential energy of a charge in the electric field:

$$W_E = Q \times \phi$$

The change in the potential energy

$$\Delta W_E = Q \times \phi_2 - Q \times \phi_1 = Q \times (\phi_2 - \phi_1)$$

$$\Delta W_E = 1 \times 10^{-9} \text{ C} \times (5 \text{ V} - 2 \text{ V}) = 1 \times 10^{-9} \text{ C} \times 3 \text{ V} = 3 \times 10^{-9} \text{ J}$$

*Important observation:* the change in the potential energy depends **ONLY** on the potential difference.
Some conclusions from the above examples

As far as only the changes in potential or potential energy are concerned, the absolute potentials of the start and end points are not important: only the difference between them.

(Compare to the mechanical potential energy: only the energy change is important)

The potential increases as the point moves towards the positive electrode (AGAINST the field lines) and decreases when it moves towards the negative electrode (ALONG the filed lines)
Voltage

Due to the relative character of potential, the most important energy characteristic of electric field is the potential difference.

The potential difference is also called the voltage $V$.

Voltage = Potential Difference

Being a potential difference, voltage is also measured in Volts (V).

If the potentials corresponding to the two different points 1 and 2 in the electric field are $\phi_1$ and $\phi_2$, the voltage $V_{21}$ between these points,

$$V_{21} = \phi_2 - \phi_1$$
Electric Potential and Voltage

$V_{21}$ is the potential energy to move the unit charge from point 1 to point 2:

$$V_{21} = \varphi_2 - \varphi_1 = E (x_2-x_1)$$

$V_{12}$ is the potential energy to move the unit charge from point 2 to point 1:

$$V_{12} = \varphi_1 - \varphi_2$$
The voltage between two charged parallel plates is 5 V. The separation between the plates is \( d = 1 \text{ mm} \). Find the electric field between the plates. Assume the field between the plates is uniform.

Solution
The electric field between the plates is uniform, hence,

\[
V = E \cdot d; \quad E = \frac{V}{d} = \frac{5V}{1\text{ mm}} = \frac{5V}{10^{-3} \text{ m}} = 5 \cdot 10^3 \text{ V/m}
\]

Answer: \( E = 5 \cdot 10^3 \text{ V/m} = 5 \text{ kV/m} \)

The electric field direction is vertically downward.
Example problem 6

The voltage between two charged parallel plates is 5 V. What energy is acquired by an electron that is moved from the bottom plate up to the top plate?

Solution
The potential energy of the electron on the bottom plate is 0 J.
The voltage of the top plate is $V = -5\text{V}$ with respect to the bottom plate.
The potential energy of the electron that moves across the voltage $V$,

$$W_E = q V = -e \cdot V = -1.6 \times 10^{-19} \text{C} \cdot (-5 \text{V}) = 8 \times 10^{-19} \text{J}$$

Answer: $W_E = 8 \times 10^{-19} \text{J}$
Uniform electric field between two charged plates is 10 V/cm. What is the voltage between two points A and B separated by the distance $d = 3$ mm?

Solution

The potentials are $\varphi_1 = E \cdot x_1; \varphi_2 = E \cdot x_2$

The voltage = potential difference is $V_{21} = \varphi_2 - \varphi_1 = E \cdot (x_2 - x_1) = E \cdot d$,

$E = 10$ V/cm = 10 V/(10$^{-2}$m) = $10^3$ V/m;

$V = 10^3$ V/m $\cdot$ 3 $\cdot$ (10$^{-3}$ m) = 3 V
The voltage between any two points in any electric field is equal to the potential difference between these two points.

\[ V_{mn} = \phi_m - \phi_n \]

Note that in the voltage indices normally, the first index is the “end” point and the second index is the “start” point.
Example problem 8

The three nodes, “1”, “2”, “3” in the amplifier circuit have the potentials
φ₁ = 3 V; φ₂ = 9 V; φ₃ = 6.5 V with respect to the reference node “0”

Question 1: find the voltages V₂₁, V₃₂, V₃₁ and V₁₃

Solution: \( V_{mn} = \phi_m - \phi_n \)

\( V_{21} = \phi_2 - \phi_1 = 9V - 3V = 6 \text{ V}; \)

\( V_{32} = \phi_3 - \phi_2 = 6.5 \text{ V} - 9V = -2.5 \text{ V}; \)

\( V_{31} = \phi_3 - \phi_1 = 6.5V - 3V = 3.5 \text{ V}; \)

\( V_{13} = \phi_1 - \phi_3 = 3V - 6.5V = -3.5 \text{ V}; \)
Example problem 8

The three nodes, “1”, “2”, “3” in the amplifier circuit have the potentials
\( \phi_1 = 3 \text{ V}; \phi_2 = 9 \text{ V}; \phi_3 = 6.5 \text{ V} \) with respect to the reference node “0”

Question 2: find the energy required to moving the charge \( Q = 2 \text{ mC} \) from the node 1 to the node 2.

Solution:

The potential energy of the charge \( Q \) at the node 1, \( W_1 = \phi_1 \times Q \);

The potential energy of the charge \( Q \) at the node 2, \( W_2 = \phi_2 \times Q \);

The change in the potential energy when the charge \( Q \) moves from the node 1 to the node 2: \( W_{21} = W_2 - W_1 = \phi_2 \times Q - \phi_1 \times Q = (\phi_2 - \phi_1) \times Q = (9 \text{ V} - 3 \text{ V}) \times 1\text{mC} = 6\text{ V} \times 1\text{mC} = 6 \text{ VmC} = 6 \text{ mJ} \)

The energy is positive, i.e. the work needs to done to move the charge
Example problem 9

In the motor driver circuit fragment on the left, the node “0” has a zero potential (the node “0” is “grounded”).

The potential of the node “1” is \( \varphi_1 = 4.5 \) V.
The potential of the node “3” is \( \varphi_3 = 9 \) V.
The voltage \( V_{21} = 4 \) V;

**Question 1**: Find the potential \( \varphi_2 \)

**Solution**: from \( V_{21} = \varphi_2 - \varphi_1 \),

\[ \varphi_2 = \varphi_1 + V_{21} = 4.5 \text{ V} + 4 \text{ V} = 8.5 \text{ V}; \]

**Question 2**: find the voltages \( V_{32} \) and \( V_{31} \)

Solution: \( V_{32} = \varphi_3 - \varphi_2 = 9\text{ V} - 8.5 \text{ V} = 0.5 \text{ V} \)

\[ V_{31} = \varphi_3 - \varphi_1 = 9\text{ V} - 4.5 \text{ V} = 4.5 \text{ V} \]