MOSFET Amplifiers and High-Frequency Performance

- In many applications, MOSFET is used as a linear (small-signal) amplifier
- A small signal equivalent circuit for MOSFET is needed to analyze the MOSFET frequency performance.

Small-Signal equivalent circuit:

- The equivalent circuit is constructed from the basic MOSFET geometry
- Resistance and capacitances are incorporated
  - Input and output signals are assumed small as compared to the steady-state DC current and voltage (operating point)

Operating point is based on the MOSFET I-Vs and circuit conditions
MOSFET amplifier: low operating frequencies

Simple FET amplifier circuit:
\[ \Delta V_L = \Delta I \times R_L \]
Voltage gain \( K_V = \Delta V_L / \Delta V_G = (\Delta I \times R_L) / \Delta V_G \)

\[ K_V = (\Delta I / \Delta V_G) \times R_L = g_m \times R_L \]

The larger the transconductance \( g_m = \Delta I / \Delta V_G \),
the higher is the voltage gain \( K_V \).
\( g_m \) is maximal in the saturation regime, i.e.
for \( V_D > V_{D_{sat}} \)

In the amplifier circuits, FETs usually work in the saturation regime,
In this regime, current DOES NOT depend on drain voltage.
Current is the function of the GATE VOLTAGE ONLY.
MOSFET operating point:
Effect of Load Resistance on the output Current amplitude

Consider the circuit with the drain bias $V_{DD} = 10$ V and the gate bias $V_{GG} = 3$ V;

1. Load resistance $R_L = 5$ kOhm
   Op. point: $V_D \approx 1.2$ V; $I_D \approx 1.8$ mA; $g_{m1} \approx (2-1.5)\text{mA}/(4-2)\text{V} \approx 0.25$ mA/V;

2. Load resistance $R_L = 0.6$ kOhm
   Op. point: $V_D \approx 6.8$ V; $I_D \approx 4.5$ mA; $g_{m2} \approx (7-2.4)\text{mA}/(4-2)\text{V} \approx 2.3$ mA/V;

In MOSFET amplifiers, the operating point is normally in the saturation region.
MOSFET Transfer Characteristics

In saturation regime, the drain current does not depend on the drain voltage.

The dependence $I_{D_{\text{sat}}}(V_G)$ is called the TRANSFER CHARACTERISTIC of MOSFET.
MOSFET amplifier: high operating frequencies

- Gate to source capacitance: $C_{gs}$
- Gate to drain capacitance: $C_{gd}$
- Drain series resistance: $r_d$
- Source series resistance: $r_s$
- Slope of $I_D$ vs $V_{ds}$: $g_{ds}$
- Input signal
- Output signal

$\Delta I = g_m V_{gs}$
Assuming $r_d, r_s = 0$
MOSFET Gain

- voltage gain

Input voltage \( v_i = v_{gs} \)
Output voltage \( v_o = \) voltage across \( g_{ds} \)

\( v_o \) can be found using superposition.
1) Assume \( v_i = 0 \).
The output voltage component is \( v_{01} \)
The output current \( i_d = i(g_{ds}) + i(C_{gd}) \)

\[
i(g_{ds}) = v_{01} \times g_{ds}; \quad i(C_{gd}) = v_{01} \times j\omega C_{gd}
\]
From this, \( i_d = v_{01} \times (g_{ds} + j\omega C_{gd}) \);
On the other hand, \( i_d = -g_m v_{gs} \)
Substituting \( i_d \), we obtain:
- \( g_m v_{gs} = v_{01} \times (g_{ds} + j\omega C_{gd}) \);

\[
v_{01} = \frac{-g_m}{g_{ds} + j\omega C_{gd}} v_{gs}
\]
MOSFET Gain

- voltage gain

2) Now, turn off the current $g_m v_{gs}$

The output voltage component is $v_{02}$

The voltage across $g_{ds} = 1/r_{ds}$:

$$v_{02} = v_i \times r_{ds} / (r_{ds} + 1/j\omega C_{gd}) =$$

$$v_i \times r_{ds} j\omega C_{gd} / (j\omega C_{gd} r_{ds} + 1) =$$

$$v_i \times j\omega C_{gd} / (j\omega C_{gd} + 1/r_{ds}) =$$

$$v_i \times j\omega C_{gd} / (j\omega C_{gd} + g_{ds});$$

$$v_{02} = \frac{j\omega C_{gd}}{g_{ds} + j\omega C_{gd}} v_{gs}$$

The total output voltage, $v_0 = v_{01} + v_{02}$:

$$v_{01} = \frac{-g_m}{g_{ds} + j\omega C_{gd}} v_{gs}$$

$$v_0 = \frac{-g_m + j\omega C_{gd}}{g_{ds} + j\omega C_{gd}} v_{gs}$$
MOSFET Gain

- voltage gain

The voltage gain:

\[ A_v = \frac{v_0}{v_{gs}} = \frac{-g_m + j\omega C_{gd}}{g_{ds} + j\omega C_{gd}} \]

The gate-drain capacitance is very low in the saturation region. Hence,

\[ A_v = \frac{v_0}{v_{gs}} \approx \frac{-g_m}{g_{ds} + j\omega C_{gd}} \]

The voltage gain decreases with frequency when \( \omega C_{gd} \approx g_{ds} \);

Corresponding characteristic frequency:

\[ f_v \approx \frac{g_{ds}}{2\pi C_{gd}} \]

At frequencies \( f > f_v \)

\[ A_v \approx \frac{-g_m}{j\omega C_{gd}} \]
MOSFET Gain

- short circuit current gain

Output current, \( i_L = i_0 = -g_m v_{gs} \)

Input current, \( i_i = i (C_{gs}) + i(C_{gs}) = v_i \times j \omega C_{gs} + v_i \times j \omega C_{gd} = v_i \times j \omega (C_{gs} + C_{gd}); \)

Also note that \( v_i = v_{gs}; \)

Therefore: \( i_i = v_i \times j \omega (C_{gs} + C_{gd}); \)

The current gain: \( A_i = \frac{i_0}{i_i} = \frac{i_L}{i_i} \)

\[
A_i = \frac{i_L}{i_i} = \frac{-g_m}{j \omega (C_{gs} + C_{gd})}
\]
MOSFET current cutoff frequency $f_T$

$f_T$ is the frequency at which the modulus of the short circuit current gain is unity.

From:

$$A_i = \frac{i_L}{i_i} = \frac{-g_m}{j\omega(C_{gs} + C_{gd})}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$f_T$ estimation

Assume that $C_{gs} + C_{gd} \sim C_i$ where $C_i = \varepsilon_iWL/d_i$ is the gate oxide capacitance.

Transconductance, $g_m$

Consider short-gate MOSFETs at high drain voltage.

In short-gate devices, the electron velocity is saturated: $I_d = q n_s v_s W$

The transconductance: $g_m = dI_d/dV_g$

$$g_m = \frac{\partial I_d}{\partial V_g} = q \frac{\partial n_s}{\partial V_g} v_s W$$
MOSFET current cutoff frequency $f_T$

Induced sheet carrier density in MOSFETs (see lecture #18):

$$n_s = \frac{c_i}{q} (V_{GS} - V_T) = \frac{c_i}{qWL} (V_{GS} - V_T)$$

where $W$ is the MOSFET width and $L$ is the gate length

From this,

$$g_m = \frac{\partial I_d}{\partial V_g} = q \frac{\partial n_s}{\partial V_g} v_s W = q \left\{ \frac{c_i}{qWL} \right\} v_s W$$

$$g_m = \frac{c_i v_s}{L}$$

The cutoff frequency becomes:

$$f_T = \frac{g_m}{2\pi \left( C_{gs} + C_{gd} \right)} = \frac{c_i v_s}{2\pi L C_i} = \frac{v_s}{2\pi L}$$
\[ f_T = \frac{g_m}{2\pi \left( C_{gs} + C_{gd} \right)} = \frac{C_{gd}v_s}{2\pi LC_i} = \frac{v_s}{2\pi L} \]

Note that, \( L/v_s = t_{tr} \), the electron transit time under the gate.

Using this, \( f_T = 1/2\pi t_{tr} \)

Practical cut-off frequency must also include the parasitic and fringing capacitances, \( C_p \), which add to the gate capacitance:

\[ f_T = \frac{g_m}{2\pi \left( C_{gs} + C_{gd} + C_p \right)} \]
**Maximum oscillation frequency, $f_{max}$**

The cutoff frequency $f_T$ is a characteristic of the intrinsic transistor (without parasitic series resistances).

$f_{max}$ is the characteristic of the extrinsic device (which includes series source, drain, input, and gate resistances, $R_s$, $R_d$, $R_i$, and $R_g$). $f_{max}$ is defined as the frequency at which the power gain of the transistor is equal to unity under optimum matching conditions for the input and output impedances.

Simplified expression for $f_{max}$ is given by:

$$f_{max} \approx \frac{f_T}{\sqrt{r + f_T \tau}} \quad \text{where} \quad r = g_{ds} (R_s + R_i + R_d) \quad \tau = 2\pi R_g C_{gd}.$$ 

In MOSFETs with small parasitic resistances, $R_s$, $R_i$ and $R_d \approx 0$,

$$f_{max} \approx \frac{f_T}{\sqrt{f_T \tau}} = \sqrt{\frac{f_T}{2\pi R_g C_{gd}}}.$$