**AC power through resistor**

Example: \( i = I_M \cos(\omega t); \ I_M = 1 \text{ A}; \ R = 10 \Omega \ \Rightarrow \ v = V_M \cos(\omega t); \ V_M = I_M \cdot R = 10 \text{ V} \)

The power dissipated in the resistor:

At any point of time, \( p(t) = v(t) \cdot i(t) = V_M \cos(\omega t) \cdot I_M \cos(\omega t) = V_M I_M \cos^2(\omega t) \)

\( p(t) \) is always positive
Instantaneous and Average ("rms") powers

RMS = “root mean square” or the square root of the arithmetic mean (average)

In terms of Joule heat dissipation, instantaneous power is equivalent to a certain DC power (i.e. time-independent).

The equivalent DC power is called Average or “rms” power.

**Instantaneous power:**

\[ p(t) = v(t) \times i(t) = V_M I_M \cos^2 (\omega t) \]
Average (“rms”) power dissipated in resistor

\[ p(t) = v(t)i(t) = V_MI_M \cos^2(\omega t) \]

\[ \cos^2(\omega t) = \frac{1}{2} [1 + \cos(2\omega t)] \]

\[ p(t) = \frac{V_MI_M}{2} + \frac{V_MI_M}{2} \cos(2\omega t) \]

\[ P_{\text{rms}} = \frac{V_MI_M}{2} \]

\[ V_{\text{eff}} = \frac{V_M}{\sqrt{2}} \]

\[ I_{\text{eff}} = \frac{I_M}{\sqrt{2}} \]
The sine voltage source with the voltage amplitude $V_M$ produces the same power as the DC battery with the voltage $V_{DC}$:

$$V_{DC} = V_M / \sqrt{2} = 0.707V_M$$

The equivalent DC voltage (producing the same Joule heat as the AC voltage) is called \textit{RMS voltage (Root Mean Square) or effective value}
Average (RMS) characteristics of sine waves

\[ V_{RMS} = \frac{V_M}{\sqrt{2}} = 0.707V_M \]

What is the amplitude of the AC source above?

\[ V_M = \frac{V_{RMS}}{0.707} = \frac{10V}{0.707} = 14.14 \text{ V} \]
AC powers in R-L-C circuits

There is a phase angle between the current and voltage:
\[ i = I_M \cos(\omega t); \quad v = V_M \cos(\omega t + \varphi); \]

The instantaneous power:
\[ p(t) = v(t) \cdot i(t) = V_M \cos(\omega t + \varphi) \cdot I_M \cos(\omega t) = V_M I_M \cos(\omega t + \varphi) \cos(\omega t) \]

\[
\cos(a) \cos(b) = \frac{1}{2} \left[ \cos(a+b) + \cos(a-b) \right] \\
a = \omega t + \varphi; \quad b = \omega t; \\
\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b) \\
a = \omega t + \varphi; \quad b = \omega t; \\
\cos(2\omega t + \varphi) = \cos(2\omega t) \cos(\varphi) - \sin(2\omega t) \sin(\varphi) \\
\]

\[
p(t) = \frac{V_M I_M}{2} \cos(\varphi) [1 + \cos(2\omega t)] - \frac{V_M I_M}{2} \sin(\varphi) \sin(2\omega t) 
\]
AC powers in R-L-C circuits

In R-L-C circuits, the current and voltage may both have a phase angle with respect to the source signal:

\[ i = I_M \cos(\omega t + \varphi_i); \quad v = V_M \cos(\omega t + \varphi_v); \]

We can offset the origin of the current and voltage waveforms by \( \varphi_i \):

\[ i = I_M \cos(\omega t); \quad v = V_M \cos(\omega t + \varphi_v - \varphi_i) = V_M \cos(\omega t + \varphi); \]

where \( \varphi = \varphi_v - \varphi_i \); we can now use the previous expressions derived for \( \varphi_i = 0 \)

\[
p(t) = \frac{V_M I_M}{2} \cos(\varphi_{vi}) \cdot [1 + \cos(2\omega t)] - \frac{V_M I_M}{2} \sin(\varphi_{vi}) \sin(2\omega t)
\]

where \( \varphi_{vi} = \varphi_v - \varphi_i \) is the phase angle between the voltage and the current
Active (average) and reactive powers in R-L-C circuits

\[ p(t) = \frac{V_M I_M}{2} \cos(\phi_v) \cdot [1 + \cos(2\omega t)] - \frac{V_M I_M}{2} \sin(\phi_v) \sin(2\omega t) \]

Active (average) power:

\[ p_A(t) = P = \frac{V_M I_M}{2} \cos(\phi_v) = V_{\text{eff}} I_{\text{eff}} \cos(\phi_v) \]
Active (average) and reactive powers in R-L-C circuits

\[ p(t) = \frac{V_M I_M}{2} \cos(\phi_{vi}) \cdot \left[1 + \cos(2\omega t)\right] - \frac{V_M I_M}{2} \sin(\phi_{vi}) \sin(2\omega t) \]

Reactive power:

\[ p_R(t) = Q = \frac{V_M I_M}{2} \sin(\phi_{vi}) = V_{eff} I_{eff} \sin(\phi_{vi}) \]

Average reactive power = 0
Active (average) and reactive powers in R-L-C circuits

\[ p(t) = P \cdot [1 + \cos(2\omega t)] - Q \sin(\phi_{vi}) \sin(2\omega t) \]

\[ P = \frac{V_M I_M}{2} \cos(\phi_{vi}) \quad Q = \frac{V_M I_M}{2} \sin(\phi_{vi}) \]

Examples:
1. Resistive circuit: \( \phi_{vi} = 0 \);
   \( P = V_M I_M / 2; \quad Q = 0; \)

2. Inductance: \( \phi_{vi} = -\pi / 2 \);
   \( P = 0; \quad Q = -V_M I_M / 2; \)

2. Capacitance: \( \phi_{vi} = \pi / 2 \);
   \( P = 0; \quad Q = +V_M I_M / 2; \)
AC powers and complex amplitudes

Given the complex current amplitude: \( \hat{I}_M = I_M e^{j\varphi_i} \)

and the complex voltage amplitude: \( \hat{V}_M = V_M e^{j\varphi_v} \)

Complex Power is defined as

\[ S = \hat{V}_{\text{eff}} \cdot \hat{I}_{\text{eff}} \]

where

\[ V_{\text{eff}} = V_M / \sqrt{2}; \quad I_{\text{eff}} = I_M / \sqrt{2} \]

\[ \hat{I}_{\text{eff}}^* = I_{\text{eff}} e^{-j\varphi_i} \quad \text{is a complex conjugate of the } I_{\text{eff}} \]

\[ S = \hat{V}_{\text{eff}} \cdot \hat{I}_{\text{eff}}^* = \frac{V_M I_M}{2} e^{j(\varphi_v - \varphi_i)} = \frac{V_M I_M}{2} \cos(\varphi_{vi}) + j \frac{V_M I_M}{2} \sin(\varphi_{vi}) \]

\[ \text{Re}(S) = P \quad \text{Im}(S) = Q \]