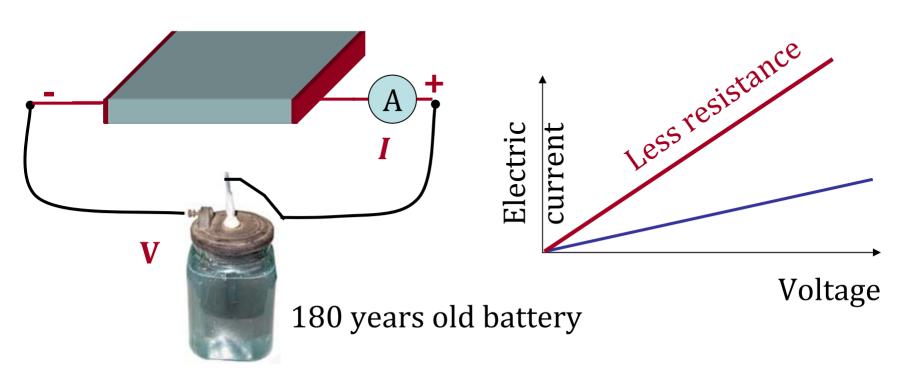# The Ohm's Law

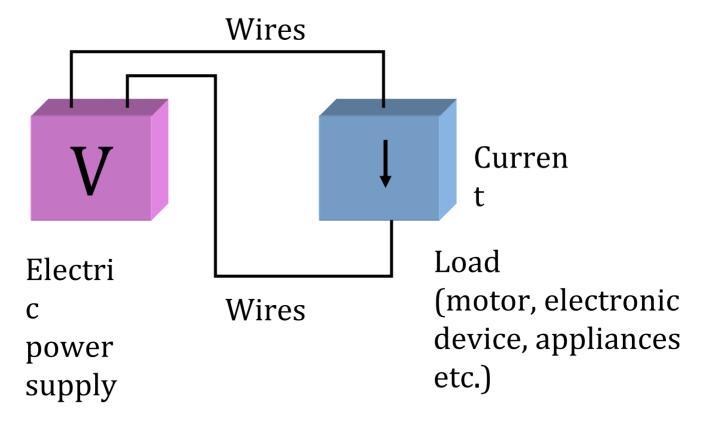George Ohm has established experimentally in 1827 the following law

$$I = \frac{1}{R} V$$

V is the voltage across a conductor
I is the current thought a conductor
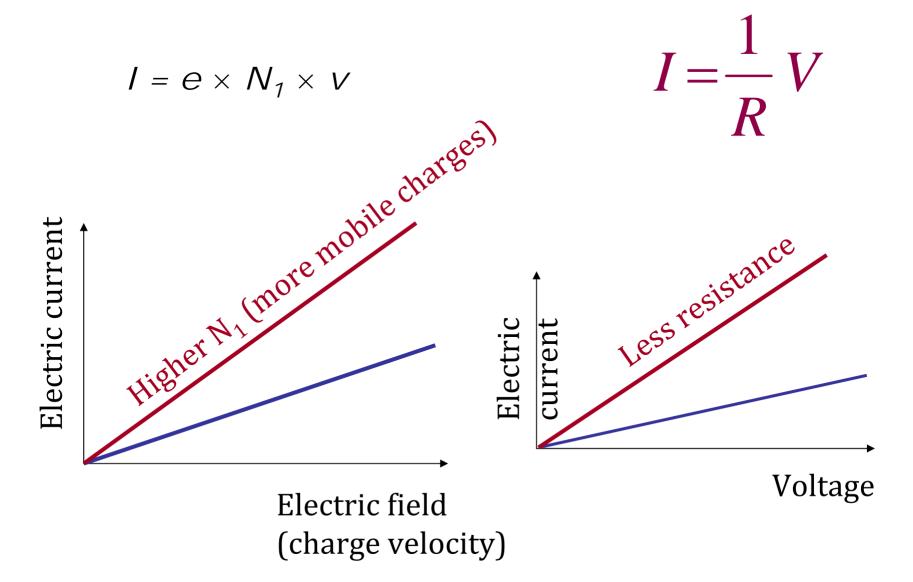1/R is a proportionality factor;
R is called "resistance"

-   **+**

A

I

V

180 years old battery

Less resistance

Electric current

Voltage

Ohm's Law

General type electric circuit:

Wires

V

↓ Curren t

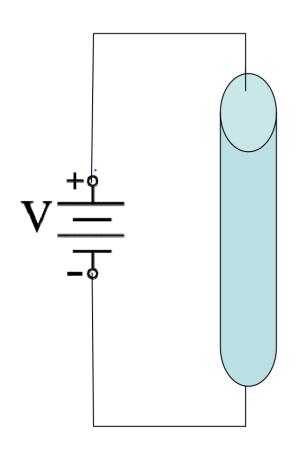Electri c power supply

Wires

Load (motor, electronic device, appliances etc.)

Given the power supply voltage V,
how much current flows through the load?

Ohm's Law and electron charge transfer rate comparison

$$I = e \times N_1 \times v$$

$$I = \frac{1}{R} V$$

Electric current — Higher N_1 (more mobile charges)

Electric field (charge velocity)

Electric current — Less resistance

Voltage

Experimental observations of Ohm's Law
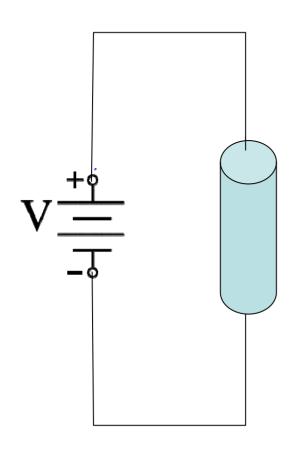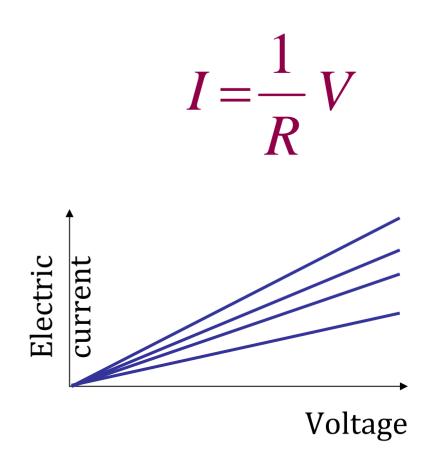
$$I = \frac{1}{R} V$$

Conductor length decreases

Current increases, i.e. R decreases

Experimental observations of Ohm's Law

$$I = \frac{1}{R} V$$

Conductor area
increases

Current increases,
i.e. R decreases

Resistance and resistivity

$$I = \frac{V}{R}$$

$$R = \rho \, \frac{L}{A}$$

L is the conductor length
(along the current direction)
A is the cross-section area
(with respect to the current direction)

ρ is called **resistivity** of the material;
ρ does not depend on the conductor shape; it depends on
mobile charge concentration and mobility in the material.

Ohm's law, resistance and resistivity - summary

$$I = \frac{V}{R}$$

Ohm's law in the form of I(V): the current through the conductor = applied voltage divided by the resistance

$$V = I \times R$$

Ohm's law in the form of I(V): the voltage needed to maintain the current I = current times resistance.

$$R = \rho \frac{L}{A}$$

A and L describe the geometry of the sample or wire.

The larger is the cross-section A the smaller is R, the higher is the current.

The longer is the wire length L, the higher is R, the smaller is the current;

$$R = \rho \frac{L}{A}$$

ρ describes the material ability to conduct the current.

The higher is ρ, the higher is R and the lower is the current

The units for resistance and resistivity

$$V = I \times R \implies R = \frac{V}{I}$$

Resistance R is measured in Ohms (Ohm, Ω)

1 Ohm is the resistance of the sample that passes the current of 1A when the voltage of 1 V is applied across it.

$$R = \rho \frac{L}{A} \implies \rho = R \frac{A}{L} \quad \left[Ohm \frac{m^2}{m} = Ohm \cdot m \right]$$

Resistivity is measured in Ohm - meters (Ohm· m)

Ohms' law using conductance and conductivity

Conductance $\boxed{G = 1/R}$

Using the conductance, the Ohm's law can be written as

$$I = \frac{1}{R} V = G \cdot V$$

Also, from $G = 1/R$ and $R = \rho \frac{L}{A}$, $G = 1/R = \frac{1}{\rho} \frac{A}{L}$

$$\sigma = \frac{1}{\rho}$$

σ is called **conductivity** of the material
(does not depend on the conductor shape)

Using conductivity, the conductance of
the sample is given by $\boxed{G = \sigma \frac{A}{L}}$

Ohm's Law using conductance - summary

The expression that relates the electric current to the applied voltage
is called the Ohm's Law (established experimentally in 1827)

$$I = G \times V, \quad where$$

$$G = \sigma \frac{A}{L} \quad is\ the\ conductance\ of\ the\ sample\ (wire),$$

$$\sigma \quad is\ the\ conductivity\ of\ the\ material$$

A is the conductor cross-section area;

L is the conductor length along the current direction

The units for conductance and conductivity

$$I = G \times V \qquad \Longrightarrow \qquad G = \frac{I}{V}$$

*Conductance is measured in **Siemens (S)***

1 S is the conductance of the sample that passes the current of 1A when the voltage of 1 V is applied across it.

$$G = \sigma \frac{A}{L} \qquad \Longrightarrow \qquad \sigma = G \frac{L}{A} \qquad \left[S \frac{m}{m^2} = S/m \right]$$

*Conductivity is measured in **Siemens per meter (S/m)***

Units and Dimensions

Charge: Coulomb [C] abs (electron charge) **e = 1.6×10^{-19} C**
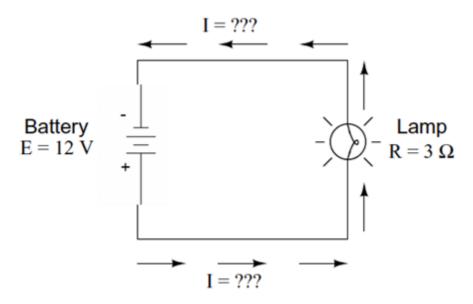
Electric current: Ampere [A]

Electric voltage: Volt [V]

Resistance: Ohm [Ohm]

Resistivity [Ohm×m]

Conductance: Siemens [S]

Conductivity: [S/m, (Ohm×m)$^{-1}$]

Example 1: the Ohm's Law

I = ???

Battery
E = 12 V

Lamp
R = 3 Ω

I = ???

What is the amount of current (I) in this circuit?

$$V = I \times R$$

$$I = V / R$$

$$I = 12V / 3\,\Omega = 4\,A$$

Note: the notation for the battery voltage is usually "V" or "E"

Resistivity of different materials

Wires are used to connect different components in the network; Wires have very low resistance

Resistors are used to dissipate the power and to change the voltage (potential).

Material	Electric Resistivity ($\times 10^{-9}$ Ohm·m)
Aluminum [Al]	**27**
Aluminum Alloy	**50**
Brass	**20 - 61**
Carbon [C]	**1.4 $\times$ 10^4**
Copper [Cu]	**17**
Copper Alloy	**17 - 490**
Gold [Au]	**24**
Iron [Fe]	**97**

Example 2

Find the resistance of a **wire** made of **copper** [Cu] (ρ = 17*10^{-9} Ohm-m). The wire is 1 m long and is 1 mm in diameter.

R = ρ *L/A

L = 1 m; D = 1mm = 10^{-3} m;

The area, A = π*D^2/4 = 3.14*(10^{-3})^2/4 = 7.85*10^{-7} m^2

The resistance

R = 17*10^{-9} Ohm*m*1m/ 7.85*10^{-7} m^2 = **2.17*10^{-2} Ohm = 21.7 mOhm**

What voltage across this wire is required to have the 100 mA current through it?

$$V = I \times R = 100\ e\text{-}3 \times 21.7e\text{-}3 = 0.00217\ V$$

Example 3

Find the resistance of a **carbon resistor**, which is 1 cm long and 0.1 mm in diameter

$R = \rho \cdot L/A$; $\rho = 1.4 \times 10^4 \cdot 10^{-9}$ Ohm*m $= 1.4 \times 10^{-5}$ Ohm*m ;

$L = 1$ cm $= 10^{-2}$ m; $A = 7.85 \cdot 10^{-9}$ m^2

$R = 1.4 \cdot 10^{-5}$ Ohm*m $\cdot 10^{-2}$ m$/ 7.85 \cdot 10^{-9}$ m$^2 =$ **17.8 Ohm**

Compare: Cu – wire, $R_1 =$ **21.7 mOhm $\approx$ 0.02 Ohm;**

Carbon-resistor, $R_2 =$ **17.8 Ohm $\approx$ 18 Ohm;**

To have the current through the Cu-wire of 0.1 A, the required voltage across the wire, $V_1 = I \cdot R_1 = 0.002$ V;

To have the same current through the Carbon-resistor, the required voltage across the resistor is $V_2 = I \cdot R_2 = 1.8$ V;