The Ohm’s Law

George Ohm has established experimentally in 1827 the following law

\[ I = \frac{1}{R} V \]

V is the voltage across a conductor
I is the current thought a conductor
1/R is a proportionality factor;
R is called “resistance”
Ohm’s Law

General type electric circuit:

Given the power supply voltage $V$, how much current flows through the load?
Ohm’s Law and electron charge transfer rate comparison

\[ I = e \times N_1 \times v \]

\[ I = \frac{1}{R} V \]

- Higher \( N_1 \) (more mobile charges)
- Less resistance
Experimental observations of Ohm’s Law

\[ I = \frac{1}{R} V \]

Conductor length decreases

Current increases, i.e. R decreases
Experimental observations of Ohm’s Law

\[ I = \frac{1}{R} V \]

Conductor area increases

Current increases, i.e. R decreases
Resistance and resistivity

\[ I = \frac{V}{R} \]

\[ R = \rho \frac{L}{A} \]

\( L \) is the conductor length (along the current direction)
\( A \) is the cross-section area (with respect to the current direction)

\( \rho \) is called **resistivity** of the material;
\( \rho \) does not depend on the conductor shape; it depends on mobile charge concentration and mobility in the material.
Ohm’s law, resistance and resistivity - summary

\[ I = \frac{V}{R} \]

Ohm’s law in the form of \( I(V) \): the current through the conductor = applied voltage divided by the resistance

\[ V = I \times R \]

Ohm’s law in the form of \( I(V) \): the voltage needed to maintain the current \( I = \) current times resistance.

\[ R = \rho \left( \frac{L}{A} \right) \]

A and L describe the geometry of the sample or wire.

The larger is the cross-section \( A \) the smaller is \( R \), the higher is the current.

The longer is the wire length \( L \), the higher is \( R \), the smaller is the current;

\[ R = \rho \left( \frac{L}{A} \right) \]

\( \rho \) describes the material ability to conduct the current.

The higher is \( \rho \), the higher is \( R \) and the lower is the current
The units for resistance and resistivity

\[ V = I \times R \quad \Rightarrow \quad R = \frac{V}{I} \]

**Resistance \( R \) is measured in **Ohms (Ohm, \( \Omega \))**

1 Ohm is the resistance of the sample that passes the current of 1A when the voltage of 1 V is applied across it.

\[ R = \rho \frac{L}{A} \quad \Rightarrow \quad \rho = R \frac{A}{L} \left[ \text{Ohm} \frac{m^2}{m} = \text{Ohm} \cdot m \right] \]

**Resistivity is measured in Ohm - meters (Ohm·m)**
Ohms’ law using conductance and conductivity

Conductance \( G = \frac{1}{R} \)

Using the conductance, the Ohm’s law can be written as

\[ I = \frac{1}{R} V = G \cdot V \]

Also, from \( G = \frac{1}{R} \) and \( R = \rho \frac{L}{A} \), \( G = \frac{1}{R} = \frac{1}{\rho \frac{L}{A}} = \frac{1}{\rho} \frac{A}{L} \)

\( \sigma = \frac{1}{\rho} \) \( \sigma \) is called conductivity of the material (does not depend on the conductor shape)

Using conductivity, the conductance of the sample is given by

\( G = \sigma \frac{A}{L} \)
Ohm’s Law using conductance - summary

The expression that relates the electric current to the applied voltage is called the Ohm’s Law (established experimentally in 1827)

\[ I = G \times V, \quad \text{where} \]
\[ G = \sigma \frac{A}{L} \quad \text{is the conductance of the sample (wire),} \]
\[ \sigma \quad \text{is the conductivity of the material} \]

\[ A \text{ is the conductor cross-section area;} \]
\[ L \text{ is the conductor length along the current direction} \]
The units for conductance and conductivity

\[ I = G \times V \quad \Rightarrow \quad G = \frac{I}{V} \]

**Conductance is measured in Siemens (S)**

1 S is the conductance of the sample that passes the current of 1A when the voltage of 1 V is applied across it.

\[ G = \sigma \frac{A}{L} \quad \Rightarrow \quad \sigma = G \frac{L}{A} \left[ S \frac{m}{m^2} = S / m \right] \]

**Conductivity is measured in Siemens per meter (S/m)**
Units and Dimensions

Charge: Coulomb [C] abs (electron charge) \( e = 1.6 \times 10^{-19} \text{ C} \)

Electric current: Ampere [A]

Electric voltage: Volt [V]

Resistance: Ohm [Ohm]

Resistivity [Ohm×m]

Conductance: Siemens [S]

Conductivity: [S/m, (Ohm×m)^{-1}]
Example 1: the Ohm’s Law

What is the amount of current (I) in this circuit?

\[ V = I \times R \]

\[ I = \frac{V}{R} \]

\[ I = \frac{12V}{3\ \Omega} = 4\ A \]

*Note: the notation for the battery voltage is usually “V” or “E”*
Resistivity of different materials

**Wires** are used to connect different components in the network; Wires have very low resistance

**Resistors** are used to dissipate the power and to change the voltage (potential).

<table>
<thead>
<tr>
<th>Material</th>
<th>Electric Resistivity ($\times 10^{-9}$ Ohm·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum [Al]</td>
<td>27</td>
</tr>
<tr>
<td>Aluminum Alloy</td>
<td>50</td>
</tr>
<tr>
<td>Brass</td>
<td>20 - 61</td>
</tr>
<tr>
<td>Carbon [C]</td>
<td>$1.4 \times 10^4$</td>
</tr>
<tr>
<td>Copper [Cu]</td>
<td>17</td>
</tr>
<tr>
<td>Copper Alloy</td>
<td>17 - 490</td>
</tr>
<tr>
<td>Gold [Au]</td>
<td>24</td>
</tr>
<tr>
<td>Iron [Fe]</td>
<td>97</td>
</tr>
</tbody>
</table>
Example 2

Find the resistance of a wire made of copper \([\text{Cu}]\) \((\rho = 17 \times 10^{-9} \text{ Ohm-m})\). The wire is 1 m long and is 1 mm in diameter.

\[
R = \rho \times \frac{L}{A}
\]

\(L = 1 \text{ m}; \, D = 1\text{mm} = 10^{-3} \text{ m};\)

The area, \(A = \pi \times \frac{D^2}{4} = 3.14 \times (10^{-3})^2 / 4 = 7.85 \times 10^{-7} \text{ m}^2\)

The resistance
\[R = 17 \times 10^{-9} \text{ Ohm} \times \frac{1 \text{m}}{7.85 \times 10^{-7} \text{ m}^2} = 2.17 \times 10^{-2} \text{ Ohm} = 21.7 \text{ mOhm}\]

What voltage across this wire is required to have the 100 mA current through it?

\[
V = I \times R = 100 \times 10^{-3} \times 21.7 \times 10^{-3} = 0.00217 \text{ V}
\]
Example 3

Find the resistance of a **carbon resistor**, which is 1 cm long and 0.1 mm in diameter

\[
R = \rho \frac{L}{A}; \quad \rho = 1.4 \times 10^4 \times 10^{-9} \text{ Ohm*m} = 1.4 \times 10^{-5} \text{ Ohm*m} ;
\]

\[
L = 1 \text{ cm} = 10^{-2} \text{m}; \quad A = 7.85 \times 10^{-9} \text{ m}^2
\]

\[
R = 1.4 \times 10^{-5} \text{ Ohm*m} \times 10^{-2} \text{ m} / 7.85 \times 10^{-9} \text{ m}^2 = 17.8 \text{ Ohm}
\]

**Compare:** Cu – wire, \( R_1 = 21.7 \text{ mOhm} \approx 0.02 \text{ Ohm} \);

Carbon-resistor, \( R_2 = 17.8 \text{ Ohm} \approx 18 \text{ Ohm} \);

**To have the current through the Cu-wire of 0.1 A, the required voltage across the wire,** \( V_1 = I \times R_1 = 0.002 \text{ V} \);

**To have the same current through the Carbon-resistor, the required voltage across the resistor is** \( V_2 = I \times R_2 = 1.8 \text{ V} \);